

Exercises for “Decision Procedures for Verification” Exercise sheet 12

In what follows we consider the theory of arrays defined in the lecture. We assume that the theory of indices \mathcal{T}_i is $LI(\mathbb{Z})$, and the theory of elements \mathcal{T}_e is $LI(\mathbb{Q})$.

Exercise 12.1: (2 P)

Which of the formulae below are in the array property fragment and which are not? Justify your answer. (The universally quantified variables i, j are sort index; the indices k, l which are not universally quantified are considered to be constants of sort index)

- (1) $\forall i (a[i + 1] > a[i])$
- (2) $\forall i (i < a[k] \rightarrow a[i] = a[k])$
- (3) $\forall i, j (l_1 \leq i \leq u_1 < l_2 \leq j \leq u_2 \rightarrow a[i] \leq a[j])$
- (3) $\forall i, j (l_1 < i \leq u_1 < l_2 \leq j \leq u_2 \rightarrow a[i] \leq a[j])$.

Exercise 12.2: (4 P)

Consider the array property formula:

$$F : \text{write}(a, l, v)[k] = b[k] \wedge b[k] \neq v \wedge a[k] = v \wedge \forall i (i \neq l \rightarrow a[i] = b[i])$$

and let F'_6 be the formula obtained (in the example presented in the lecture) by applying Steps 1–6 to F , after simplification.

$$F'_6 : \quad a'[k] = b[k] \wedge b[k] \neq v \wedge a[k] = v \wedge a[\lambda] = b[\lambda] \wedge (k \neq l \rightarrow a[k] = b[k]) \\ \wedge a'[l] = v \wedge a[\lambda] = a'[\lambda] \wedge (k \neq l \rightarrow a[k] = a'[k]) \wedge \lambda \neq k \wedge \lambda \neq l.$$

Check the satisfiability of F'_6 w.r.t. $\mathcal{T} = UIF_{\{a, b, a'\}} \cup \mathcal{T}_i \cup \mathcal{T}_e$ using one of the versions of the $DPLL(\mathcal{T})$ procedure presented in the class. For theory reasoning in \mathcal{T} use the Nelson-Oppen procedure.

Exercise 12.3: (4 P)

Consider the following array property formula:

$$F : \forall i (l \leq i \leq u \rightarrow a[i] = b[i]) \wedge \neg \forall i (l \leq i \leq u + 1 \rightarrow \text{write}(a, u + 1, b[u + 1])[i] = b[i])$$

Apply to the formula F the Steps 1–6 of the transformation procedure for formulae in the array property fragment presented in the lecture.

Please submit your solution until Friday, January 27, 2012 at 17:00 by e-mail to sofronie@uni-koblenz.de with the keyword "Homework DP" in the subject.

You can send updates or additions to your solution before Sunday, January 29, 2012 at 17:00.

Joint solutions prepared by up to two persons are allowed.

Please do not forget to write your name on your solution!