Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans **October 25, 2011**

Exercises for "Decision Procedures for Verification"
Exercise sheet 2

Exercise 2.1: (*2 P*)
Prove Prop. 1.4: If N is a set of propositional formulas, then $N \models F$ if and only if $N \cup \{\neg F\}$ is unsatisfiable.

(A set of propositional formulas is unsatisfiable, if and only if for every valuation $\mathcal{A}$ there is a formula G in the set such that $\mathcal{A} \not\models G$.)

Exercise 2.2: (*2 P*)
Let F be a formula, P a propositional variable not occurring in F, and F' a subformula of F. Prove: The formula $F[P] \wedge (P \leftrightarrow F')$ is satisfiable if and only if $F[F']$ is satisfiable.

Exercise 2.3: (*4 P*)
Let F be the following formula:

$$\neg[((Q \wedge \neg P) \wedge \neg(Q \wedge R)) \rightarrow (Q \wedge (Q \rightarrow P) \wedge \neg P)] \wedge (P \vee R)$$

(1) Compute the negation normal form (NNF) F' of F.

(2) Convert F' to CNF using the satisfiability-preserving transformation described in the lecture.

Exercise 2.4: (*2 P*)
Determine the polarity of the following subformulae of

$$F = \neg((\neg(P \wedge \neg Q)) \vee (R \vee \neg S)) \vee (U \wedge V)$$

(1) $(P \wedge \neg Q)$

(2) Q

(3) $(R \vee \neg S)$

(4) S

(5) V

(6) $((\neg(P \wedge \neg Q)) \vee (R \vee \neg S))$

Supplementary exercises:
(submission not obligatory, will be discussed in the exercise session)

Exercise 2.5: *(2 P)*

Consider the formulae $F_n = \bigvee_{i=1}^{n} (Q_i \wedge R_i)$ for $n \in \mathbb{N}$.
As a function of n, how many clauses are in:

(1) the CNF formula F' constructed using the distributivity of disjunctions over conjunctions?
(2) the CNF formula F'' obtained using the satisfiability-preserving translation to clause form?

(3) For which n is the first approach better?

Reminder: The structural induction principle (for propositional logic).

Let $\mathcal{B}$ be a property of formulae in propositional logic. Assume that the following hold:

- for every propositional variable $P \in \Pi$, P has property $\mathcal{B}$;
- $\bot$ and $\top$ have property $\mathcal{B}$;
- if $F = F_1$ $\texttt{op}$ F_2 for $\texttt{op} \in \{\vee, \wedge, \rightarrow, \leftrightarrow\}$ and if both F_1 and F_2 have property $\mathcal{B}$ then F has property $\mathcal{B}$;
- if $F = \neg F_1$ and F_1 has property $\mathcal{B}$ then F has property $\mathcal{B}$.

Then property $\mathcal{B}$ holds for all Π-formulae.

Exercise 2.6: *(4 P)*
Let F be a formula containing neither $\rightarrow$ nor $\leftrightarrow$, P a propositional variable not occurring in F, and F' a subformula of F. Prove:

- If F' has positive polarity in F then $F[F']$ is satisfiable if and only if $F[P] \wedge (P \rightarrow F')$ is satisfiable.
- If F' has negative polarity in F then $F[F']$ is satisfiable if and only if $F[P] \wedge (F' \rightarrow P)$ is satisfiable.

Please submit your solution until Friday, October 28, 2011 at 17:00. Joint solutions prepared by up to two persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework DP" in the subject.
- Put it in the box in front of Room B 225 (if you prefer to submit the written exercise like this please tell me such that I can prepare such a box).