Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Decision Procedures for Verification"
Exercise sheet 3

Exercise 3.1: (*3 P*)
Use the resolution calculus to prove that the following set of clauses is unsatisfiable:

(1)	$\neg P \lor \neg Q \lor R$
(2)	$\neg P \lor \neg Q \lor S$
(3)	P
(4)	$\neg S \lor \neg R$
(5)	Q

Exercise 3.2: (*1 P*)
Find a total ordering on the propositional variables A, B, C, D, E, such that the associated clause ordering $\succ_C$ orders the clauses like this:

$$B \lor C \succ_C A \lor A \lor \neg C \succ_C C \lor E \succ_C C \lor D \succ_C \neg A \lor D \succ_C \neg E.$$

Exercise 3.3: (*4 P*)
Let N be the following set of clauses:

(1)	$\neg P_3 \lor P_1 \lor P_1$
(2)	$\neg P_2 \lor P_1$
(3)	$P_4 \lor P_4$
(4)	P_4
(5)	$P_3 \lor \neg P_2$
(6)	$P_4 \lor P_3$

(1) Let $\succ$ be the ordering on propositional variables defined by $P_4 \succ P_3 \succ P_2 \succ P_1$. Sort the clauses in N according to $\succ_C$. Which literals are maximal in the clauses of N?
(2) Define a selection function S such that N is saturated under $Res_S^{\succ}$.
(3) Construct a model of N using the canonical construction presented in the lecture.

Exercise 3.4: (*2 P*)
Use a DPLL procedure to find a model of each of the following formulae, or prove that the particular formula has no model:

(1) $(P \lor \neg Q) \land (\neg P \lor Q) \land (Q \lor \neg R) \land (\neg Q \lor \neg R)$
(2) $(P \lor Q \lor \neg R) \land (P \lor \neg Q) \land (P \lor Q \lor R) \land (R \lor Q) \land (R \lor \neg Q) \land (\neg P \lor \neg R) \land \neg U$

Supplementary exercises:
(submission not obligatory, will be discussed in the exercise session)

Reminder: The structural induction principle (for propositional logic).

Let $\mathcal{B}$ be a property of formulae in propositional logic. Assume that the following hold:

- for every propositional variable $P \in \Pi$, P has property $\mathcal{B}$;
- $\bot$ and $\top$ have property $\mathcal{B}$;
- if $F = F_1 \; \mathsf{op} \; F_2$ for $\mathsf{op} \in \{\vee, \wedge, \rightarrow, \leftrightarrow\}$ and if both F_1 and F_2 have property $\mathcal{B}$ then F has property $\mathcal{B}$;
- if $F = \neg F_1$ and F_1 has property $\mathcal{B}$ then F has property $\mathcal{B}$.

Then property $\mathcal{B}$ holds for all Π-formulae.

Exercise 3.5: *(2 P)*
Let F be a formula and F' a subformula of F, occurring only once, at a given position in F. We will denote this by $F = F[F']$. Let F'' be another formula. We denote by $F[F'']$ the formula obtained from F by replacing F' by F''. Let $\mathcal{A}$ be a valuation. Assume that $\mathcal{A}(F_1) \leq \mathcal{A}(F_2)$. Prove:

(1) If F' has positive polarity in F then $\mathcal{A}(F[F']) \leq \mathcal{A}(F[F''])$.
(2) If F' has negative polarity in F then $\mathcal{A}(F[F']) \geq \mathcal{A}(F[F''])$.

Hint: Use structural induction for proving both statements at the same time.

Exercise 3.6: *(2 P)*
Let F be a formula containing neither $\rightarrow$ nor $\leftrightarrow$, P a propositional variable not occurring in F, and F' a subformula of F. Prove:

- If F' has positive polarity in F then $F[F']$ is satisfiable if and only if $F[P] \wedge (P \rightarrow F')$ is satisfiable.

- If F' has negative polarity in F then $F[F']$ is satisfiable if and only if $F[P] \wedge (F' \rightarrow P)$ is satisfiable.

Exercise 3.7: *(5 P)*
Let N be a set of clauses in propositional logic with the property that each clause consists of two literals. Prove that the satisfiability of N can be checked in polynomial time in the size of N.

Please submit your solution until Friday, November 4, 2011 at 17:00 (by e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework DP" in the subject). Joint solutions prepared by up to two persons are allowed. Please do not forget to write your name on your solution!