Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Decision Procedures for Verification"
Exercise sheet 4

Exercise 4.1: (*2 P*)
A propositional Horn clause is a clause which has at most one positive literal.
(*Example:* $\neg P \vee Q \vee \neg R$, $\neg P \vee \neg R$ and Q are Horn clauses,
whereas $\neg P \vee Q \vee R$ and $Q \vee R$ are not Horn clauses.)
Prove: Every set H of clauses with the following properties:

(i) H consists only of Horn clauses;
(ii) Every clause in H contains at least one negative literal;

is satisfiable.

Exercise 4.2: (*2 P*)
Let H be a set of Horn clauses. The size of H is the number of all literals which occur in H. Prove that the resolution calculus $\mathsf{Res}_S^{\succ}$ can check the satisfiability of H in polynomial time in the size of H. (Find the ordering/selection function for which this is possible.)

Exercise 4.3: (*2 P*)
Let N be a set of clauses in propositional logic with the property that each clause consists of two literals. Prove that the satisfiability of N can be checked in polynomial time in the size of N.

(*Hint (a possible solution):*

- *How many clauses consisting of two literals (over a* finite *set of propositional variables $\Pi = \{P_1, \dots, P_n\}$) exist?*
- *Analyze the form of possible resolution inferences from N.*
- *Let N be a set of clauses in propositional logic with the property that each clause consists of two literals. Show that*
  - *If N is satisfiable then we cannot generate from N, using the resolution calculus, both $P \vee P$ and $\neg P \vee \neg P$ for some propositional variable P.*
  - *If we cannot generate from N, using the resolution calculus, both $P \vee P$ and $\neg P \vee \neg P$ for some propositional variable P then N is satisfiable.*
- *Show that the number of inferences by resolution from N which yield different clauses is polynomial in the size of N and in the size of Π. Infer that the satisfiability of N can be checked in polynomial time in the size of N.)*

Exercise 4.4: *(2 P)*
Let $\Sigma = (\Omega, \Pi)$ be a signature, where $\Omega = \{f/2,\ g/1,\ a/0,\ b/0\}$ and $\Pi = \{p/2\}$; let X be the set of variables $\{x, y, z\}$. Which of the following expressions are terms over Σ and X, which are atoms/literals/clauses/formulae, which are neither?

(a) $\neg p(g(a), f(x, y))$
(b) $f(x, x) \approx x$
(c) $p(f(x, a), x) \vee p(a, b)$
(d) $p(\neg g(x), g(y))$
(e) $\neg p(f(x, y))$
(f) $p(a, b) \wedge p(x, y) \wedge y$
(g) $\exists y(\neg p(f(y, y), y))$
(h) $\forall x \forall y(g(p(x, y)) \approx g(x))$

Exercise 4.5: *(2 P)*
Compute the results of the following substitutions:

(a) $f(g(x), x)[g(a)/x]$
(b) $p(f(y, x), g(x))[x/y]$
(c) $\forall y(p(f(y, x), g(y)))[x/y]$
(d) $\forall y(p(f(y, x), x))[y/x]$
(e) $\forall y(p(f(z, g(y)), g(x)) \vee \exists z(g(z) \approx y))[g(b)/z]$
(f) $\exists y\big(f(x, y) \approx x \rightarrow \forall x(f(x, y) \approx x)\big)[g(y)/y, g(z)/x]$

Please submit your solution until Friday, November 11, 2011 at 17:00 by e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework DP" in the subject.

Joint solutions prepared by up to two persons are allowed.
Please do not forget to write your name on your solution!