Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Decision Procedures for Verification"
Exercise sheet 7

Exercise 7.1: *(3 P)*
Consider the following formulae:

- $F_1 := \forall x(S(x) \rightarrow \exists y(R(x,y) \land P(y)))$
- $F_2 := \forall x(P(x) \rightarrow Q(x))$
- $F_3 := \exists x S(x)$
- $G := \exists x \exists y(R(x,y) \land Q(y))$

Use resolution to prove that $\{F_1, F_2, F_3\} \models G$.

Exercise 7.2: *(3 P)*
Let $\succ$ be a total and well-founded ordering on ground atoms such that, if the atom A contains more symbols than B, then $A \succ B$. Let N be the following set of clauses:

$$\neg q(z,z)$$
$$\neg q(f(x),y) \lor q(f(f(x)),y) \lor p(x)$$
$$\neg p(a) \lor \neg p(f(a)) \lor q(f(a), f(f(a)))$$
$$p(f(x)) \lor p(g(y))$$
$$\neg p(g(a)) \lor p(f(f(a)))$$

(a) Which literals are maximal in the clauses of N?
(b) Define a selection function S such that N is saturated under $Res_S^{\succ}$. Justify your choice.

Exercise 7.3: *(2 P)*
Consider the notion of redundancy defined in the slides from November 28 on page 32.

Assume that $S \succ P \succ Q \succ R$. Is the clause $C = \neg P \lor S$ redundant w.r.t. the set of clauses

$$N = \{\neg Q \lor \neg P, R \lor P, Q \lor S\}?$$

Justify your answer.

Exercise 7.4: *(2 P)*
Redundant clauses remain redundant, if the theorem prover derives new clauses and adds them to the current set of clauses. Prove:

If N and M are sets of clauses and $N \subseteq M$, then $\mathrm{Red}(N) \subseteq \mathrm{Red}(M)$.

Supplementary exercise:

Exercise 7.5: (*7 P*)
Let $\Sigma = (\{c_1/0, \dots, c_n/0, f_1/1, \dots, f_n/1\}, \Pi)$ be a signature. Consider the following classes of clauses:

- G (denoted in the lecture also $G(c_1, \dots, c_n)$) is the class of all ground clauses in the signature Σ which do not contain any occurrence of a unary function symbol.
- V (denoted in the lecture also $V(x, c_1, \dots, c_n)$) is the class of all clauses with one variable (x) in the signature Σ which do not contain any occurrence of a unary function symbol.
- G_f (denoted in the lecture also $G(c_1, \dots, c_n, f_k(c_j))$) is the class of all ground clauses in the signature Σ which contain at least one occurrence of a unary function symbol (having as argument a constant); no nested applications of unary function symbols are allowed.

  Example: Assume $p/3.q/2 \in \Pi$ Then:
  $C_1 : p(c_1, c_2, c_3) \vee \neg q(c_2, c_1)$ is in G but is not in G_f
  $C_2 : q(c_1, c_2) \vee \neg q(f_1(c_3), c_4) \in G_f$
  $C_3 : q(c_1, c_2) \vee \neg q(f_1(c_3), f_2(f_3(c_4))) \notin G_f$.

- V_f (denoted in the lecture also $V(x, c_1, \dots, c_n, f_j(x))$) is the class of all ground clauses in the signature Σ which contain only one variable (x), at least one occurrence of a unary function symbol (having as argument the variable x), no occurrences of terms of the form $f_k(c_j)$; in addition no nested applications of unary function symbols are allowed.

  Example: Assume $p/3.q/2 \in \Pi$ Then:
  $C'_1 : p(x, c_2, x) \vee \neg q(c_2, c_1)$ is in V but is not in V_f
  $C'_2 : q(x, c_2) \vee \neg q(f_1(c_3), x) \notin V_f$
  $C'_3 : q(c_1, x) \vee \neg q(x, f_2(f_3(x))) \notin V_f$ $C'_4 : p(x, c_2, x) \vee \neg p(c_2, x, f(x)) \in V_f$.

Consider a term ordering $\succ$ in which $f(t) \succ t$ for every term t and terms containing function symbols of arity 1 are larger than those who do not. Consider the general ordered resolution calculus $\mathsf{Res}^{\succ}$. Prove that in this calculus:

(1) The resolvent of a clause in G_f and a clause in V_f is a clause in G_f of G.

(2) The resolvent of two clauses in V_f is a clause in G, G_f, V or V_f.

Please submit your solution until Friday, December 2, 2011 at 17:00 by e-mail to `sofronie@uni-koblenz.de` with the keyword “Homework DP” in the subject.

Joint solutions prepared by up to two persons are allowed.
Please do not forget to write your name on your solution!