Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Decision Procedures for Verification"
Exercise sheet 13

Exercise 13.1:
Let $\mathcal{T}$ be the combination of $LI(\mathbb{Q})$ (linear arithmetic over $\mathbb{Q}$) and UIF_Σ, the theory of uninterpreted function symbols in the signature $\Sigma = \{\{f/1, g/2\}, \emptyset\}$.

Check the satisfiability of the following ground formula w.r.t. $\mathcal{T}$ using the deterministic version of the Nelson-Oppen procedure (after purifying the formulae check for entailment of equalities between shared constants and propagate the entailed equalities):

$$\phi = (c + f(d) \approx e \wedge d \approx d' \wedge f(e) \approx c + f(d') \wedge f(f(c + f(d))) \not\approx e).$$

For reasoning in UIF_Σ use the graph-based method for computing the congruence closure presented in the class. For reasoning in $LI(\mathbb{Q})$ use the Fourier-Motzkin algorithm.

Exercise 13.2:
Check the satisfiability w.r.t. $\mathcal{T} = LI(\mathbb{Q})$ of the following set of ground clauses using the "lazy" approach to SMT presented in the class.

$$(\neg(0 \leq x) \vee \neg(y \leq z)) \quad \wedge \quad (\neg(z \leq x + y) \vee (y \leq z)) \quad \wedge \quad (\neg(0 \leq y) \vee (0 \leq x)) \quad \wedge \quad (z \leq x + y)$$

For theory reasoning in $LI(\mathbb{Q})$ use the Fourier-Motzkin algorithm.

Supplementary exercises:

Exercise 13.3:
Let $\mathcal{T}$ be a theory with signature $\Sigma = (\Omega, \Pi)$ and $\mathsf{Mod}(\mathcal{T})$ be its class of models.
Show that if $\mathsf{Mod}(\mathcal{T})$ is closed under products then $\mathcal{T}$ is Π-convex.

Exercise 13.4:
We say that a theory $\mathcal{T}$ is *stably infinite* if for every quantifier-free formula ϕ, ϕ is satisfiable in $\mathcal{T}$ iff ϕ is satisfiable in a (countably) infinite model of $\mathcal{T}$.
Let $\mathcal{T}_1, \mathcal{T}_2$ be stably infinite theories with disjoint signatures. Prove that their combination $\mathcal{T}_1 \cup \mathcal{T}_2$ is stably infinite.

Please submit your solution until Monday, January 28, 2013 at 9:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution. Submission possibilities:

- By e-mail to `mbender@uni-koblenz.de` with the keyword "Homework DP" in the subject.
- Put it in the box in front of Room B 222.