Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans
Dipl.-Inform. Markus Bender

October 30, 2012

Exercises for "Decision Procedures for Verification"
Exercise sheet 3

Exercise 3.1:
Determine the polarity of the following subformulae of

$$F = \neg((\neg(P \wedge \neg Q)) \vee (R \vee \neg S)) \vee (U \wedge V)$$

(1) $(P \wedge \neg Q)$
(2) $(R \vee \neg S)$
(3) $((\neg(P \wedge \neg Q)) \vee (R \vee \neg S))$
(4) Q
(5) S
(6) V

Exercise 3.2:
Let F be the following formula in NNF:

$$[((Q \wedge \neg P) \wedge (\neg Q \vee \neg R)) \vee (Q \wedge (\neg Q \vee P) \wedge \neg P)] \wedge (P \vee R).$$

Convert F to CNF using the *satisfiability-preserving transformation* described in the lecture.

Exercise 3.3:
Consider the formulae $F_n = \bigvee_{i=1}^{n} (Q_i \wedge R_i)$ for $n \in \mathbb{N}$.
In the previous exercise we saw that the CNF formula F'_n constructed using the distributivity of disjunctions over conjunctions from F has 2^n clauses.

- As a function of n, how many clauses are in the CNF formula F''_n obtained using the structure-preserving translation to clause form?
- For which n is the first approach better?

Exercise 3.4:
Use the resolution calculus to prove that the following set of clauses is unsatisfiable:

(1)	$\neg Q \vee \neg R \vee P$
(2)	$\neg Q \vee \neg R \vee S$
(3)	$Q \vee \neg R$
(4)	$\neg S \vee \neg P$
(5)	R

Exercise 3.5:
Find a total ordering on the propositional variables A, B, C, D, E, such that the associated clause ordering $\succ_C$ orders the clauses like this:

$$B \vee C \succ_C A \vee A \vee \neg C \succ_C C \vee E \succ_C C \vee D \succ_C \neg A \vee D \succ_C \neg E.$$

Exercise 3.6:
Let N be the following set of clauses:

$$\begin{array}{cc} (1) & \neg P_3 \vee P_1 \vee P_1 \\ (2) & \neg P_2 \vee P_1 \\ (3) & P_4 \vee P_4 \\ (4) & P_4 \\ (5) & P_3 \vee \neg P_2 \\ (6) & P_4 \vee P_3 \end{array}$$

(1) Let $\succ$ be the ordering on propositional variables defined by $P_4 \succ P_3 \succ P_2 \succ P_1$. Sort the clauses in N according to $\succ_C$. Which literals are maximal in the clauses of N?

(2) Can you construct a model of N using the canonical construction presented in the lecture.

Supplementary exercise (continuation from sheet 2)

Let F be a formula, P a propositional variable not occurring in F, and F' a subformula of F.

We will write F also as $F[F']$ in order to emphasize that F' occurs in F. Let $F[P]$ be the formula obtained from F by replacing the subformula F' with the propositional variable P.

Prove:
The formula $F[P] \wedge (P \leftrightarrow F')$ is satisfiable if and only if $F[F']$ is satisfiable.

Hint: *The original property itself is to weak and cannot be proven by induction, due to the fact that we cannot assume satisfiability for $\neg G$ in the induction step $F = \neg G$.*

A way to still prove this property is to prove the following two things:

- *Let $\mathcal{A} : \Pi \to \{0, 1\}$ such that $\mathcal{A}(P) = \mathcal{A}(F')$, then $\mathcal{A}(F[P]) = \mathcal{A}(F[F'])$.*
- *$F[P] \wedge (P \leftrightarrow F')$ is satisfiable if and only if $F[F']$ is satisfiable.*

Supplementary exercise

Let F be a formula containing neither $\to$ nor $\leftrightarrow$, P a propositional variable not occurring in F, and F' a subformula of F.
We will write F also as $F[F']$ in order to emphasize that F' occurs in F. Let $F[P]$ be the formula obtained from F by replacing the subformula F' with the propositional variable P.

Prove:

(1) If F' has positive polarity in F then $F[F']$ is satisfiable if and only if $F[P] \wedge (P \to F')$ is satisfiable.

(2) If F' has negative polarity in F then $F[F']$ is satisfiable if and only if $F[P] \wedge (F' \rightarrow P)$ is satisfiable.

Hint: *You can prove by simultaneous structural induction that for every valuation $\mathcal{A}$ the following hold (assuming that $0 \leq 0, 0 \leq 1, 1 \leq 1$):*

- *If F' has positive polarity in F and $\mathcal{A}(P) \leq \mathcal{A}(F')$ then $\mathcal{A}(F[P]) \leq \mathcal{A}(F[F'])$.*
- *If F' has negative polarity in F and $\mathcal{A}(F') \leq \mathcal{A}(P)$ then $\mathcal{A}(F[P]) \leq \mathcal{A}(F[F'])$.*

and use this result in order to prove (1) and (2).

Please submit your solution until Monday, November 5, 2012 at 09:00 . Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `mbender@uni-koblenz.de` with the keyword "Homework DP" in the subject.
- Put it in the box in front of Room B 222.