Universität Koblenz-Landau

FB 4 Informatik

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November 5, 2012

Exercises for "Decision Procedures for Verification"
Exercise sheet 4

Exercise 4.1:

Assume $P \succ Q \succ R$. Let N_1 be the following set of clauses:

$$\begin{array}{ll} (C_1) & \neg R \vee P \\ (C_2) & \neg Q \vee \neg P \\ (C_3) & Q \\ (C_4) & R \vee P \end{array}$$

Use the ordered resolution calculus $\mathsf{Res}^{\succ}$ described in the lecture for checking the satisfiability of the set N_1 of clauses.

Exercise 4.2:

Assume $S \succ P \succ Q \succ R$. Let N_2 be the following set of clauses:

$$\begin{array}{ll} (C_2) & \neg Q \vee \neg P \\ (C_4) & R \vee P \\ (D_1) & Q \vee S \\ (D_2) & \neg Q \vee \neg S \end{array}$$

(1) Define a selection function S such that this set of clauses is saturated w.r.t. the ordered resolution calculus with selection $\mathsf{Res}_S^{\succ}$. Justify your choice.

(2) Sort the clauses according to $\succ_C$.

(3) Construct a model of N_2 using the canonical construction presented in the lecture.

Exercise 4.3:

A propositional Horn clause is a clause which has at most one positive literal.
(*Example:* $\neg P \vee Q \vee \neg R$, $\neg P \vee \neg R$ and Q are Horn clauses,
whereas $\neg P \vee Q \vee R$ and $Q \vee R$ are not Horn clauses.)

Prove: Every set H of clauses with the following properties:

(i) H consists only of Horn clauses;

(ii) Every clause in H contains at least one negative literal;

is satisfiable.

Exercise 4.4:
Use a DPLL procedure to find a model of each of the following formulae, or prove that the particular formula has no model:

(1) $(P \vee \neg Q) \wedge (\neg P \vee Q) \wedge (Q \vee \neg R) \wedge (\neg Q \vee \neg R)$
(2) $(P \vee Q \vee \neg R) \wedge (P \vee \neg Q) \wedge (P \vee Q \vee R) \wedge (R \vee Q) \wedge (R \vee \neg Q) \wedge (\neg P \vee \neg R) \wedge \neg U$

Supplementary exercise

Exercise 4.5:
Let H be a set of Horn clauses. The size of H is the number of all literals which occur in H.

- Prove that the resolution calculus $\mathsf{Res}_S^{\succ}$ can check the satisfiability of H in polynomial time in the size of H. (Find the ordering/selection function for which this is possible.)
- Can you give an algorithm for check the satisfiability of H in time linear in the size of H?

Exercise 4.6:
Let N be a set of clauses in propositional logic with the property that each clause consists of two literals. Prove that the satisfiability of N can be checked in polynomial time in the size of N.

(*Hint (a possible solution):*

- *How many clauses consisting of two literals (over a* finite *set of propositional variables* $\Pi = \{P_1, \dots, P_n\}$*) exist?*
- *Analyze the form of possible resolution inferences from* N*.*
- *Let* N *be a set of clauses in propositional logic with the property that each clause consists of two literals. Show that*
  - *If* N *is satisfiable then we cannot generate from* N*, using the resolution calculus, both* $P \vee P$ *and* $\neg P \vee \neg P$ *for some propositional variable* P*.*
  - *If we cannot generate from* N*, using the resolution calculus, both* $P \vee P$ *and* $\neg P \vee \neg P$ *for some propositional variable* P *then* N *is satisfiable.*
- *Show that the number of inferences by resolution from* N *which yield different clauses is polynomial in the size of* N *and in the size of* Π*. Infer that the satisfiability of* N *can be checked in polynomial time in the size of* N*.)*

Please submit your solution until Monday, November 12, 2012 at 09:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `mbender@uni-koblenz.de` with the keyword "Homework DP" in the subject.
- Put it in the box in front of Room B 222.