Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Decision Procedures for Verification"
Exercise sheet 5

Exercise 5.1:
Let $\Sigma = (\Omega, \Pi)$ be a signature, where $\Omega = \{f/3,\ g/1,\ a/0,\ b/0\}$ and $\Pi = \{p/2\}$; let X be the set of variables $\{x, y, z\}$. Which of the following expressions are terms over Σ and X, which are atoms/literals/clauses/formulae (in first-order logic with equality), which are neither?

(a) $\neg p(g(a), f(x, y, g(a)))$

(b) $f(x, x, x) \approx x$

(c) $p(f(x, x, a), x) \vee p(a, b)$

(d) $p(\neg g(x), g(y))$

(e) $\neg p(f(x, y, y))$

(f) $\neg p(f(x, y), y) \vee p(x, y)$

(g) $p(a, b) \wedge p(x, y) \wedge y$

(h) $\exists y (\neg p(f(y, y, y), y))$

(i) $\forall x \forall y (f(p(x, y), x, x) \approx g(x))$

Exercise 5.2:
Let $\Sigma = (S, \Omega, \Pi)$ be a many-sorted signature, where $S = \{\mathsf{int}, \mathsf{list}\}$, $\Omega = \{\mathsf{cons}, \mathsf{car}, \mathsf{cdr}, \mathsf{nil}, b\}$ and $\Pi = \{p\}$ with the following arities:

$a(\mathsf{cons}) = \mathsf{int}, \mathsf{list} \to \mathsf{list} \qquad a(\mathsf{car}) = \mathsf{list} \to \mathsf{int} \quad a(\mathsf{cdr}) = \mathsf{list} \to \mathsf{list}$

$a(\mathsf{nil}) = \to \mathsf{list}$ (i.e. nil is a constant of sort list)

$a(b) = \to \mathsf{int}$ (i.e. b is a constant of sort int)

$a(p) = \mathsf{int}, \mathsf{list}.$

Let X_{int} be the set of variables of sort int containing $\{i, j, k\}$, and let X_{list} be the set of variables of sort list containing $\{x, y, z\}$. Let $X = \{X_{\mathsf{int}}, X_{\mathsf{list}}\}$. Which of the following expressions are terms over Σ and X, which are atoms/literals/clauses/formulae[1], which are neither?

(a) $\mathsf{cons}(\mathsf{cons}(b, \mathsf{nil}), \mathsf{nil})$

(b) $\mathsf{cons}(b, \mathsf{cons}(b, \mathsf{nil}))$

(c) $\neg p(b, \mathsf{cons}(b, \mathsf{cons}(b, \mathsf{nil})))$

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[1] In first-order logic with equality, where equality between terms of sort int is $\approx_i$ and equality between terms of sort list is $\approx_l$.

(d) $\neg p(\mathsf{cons}(b, \mathsf{nil}), \mathsf{cons}(b, \mathsf{cons}(b, \mathsf{nil})))$

(e) $\mathsf{cons}(b, \mathsf{cons}(b, \mathsf{nil})) \approx_l \mathsf{cons}(\mathsf{cons}(x, b), \mathsf{nil})$

(f) $\mathsf{cons}(i, \mathsf{cons}(b, \mathsf{nil})) \approx j$

(g) $p(\neg \mathsf{car}(x), x)$

(h) $\neg p(\mathsf{car}(x), x) \vee p(j, \mathsf{cons}(j, x))$

(i) $\neg p(b, x) \vee p(b, \mathsf{cons}(b, x)) \vee b$

(j) $\forall i : \mathsf{int}, \forall x : \mathsf{list}\ (\mathsf{cons}(\mathsf{car}(x), \mathsf{cdr}(x)) \approx_l x)$

(k) $\exists i : \mathsf{int}, \forall y : \mathsf{list}\ (\mathsf{cons}(b, p(x, y)) \approx_l \mathsf{cdr}(y))$

Exercise 5.3:
Compute the results of the following substitutions:

(a) $f(g(x), x)[g(a)/x]$

(b) $p(f(y, x), g(x))[x/y]$

(c) $\forall y(p(f(y, x), g(y)))[x/y]$

(d) $\forall y(p(f(y, x), x))[y/x]$

(e) $\forall y(p(f(z, g(y)), g(x)) \vee \exists z(g(z) \approx y))[g(b)/z]$

(f) $\exists y\big(f(x, y) \approx x \rightarrow \forall x(f(x, y) \approx x)\big)[g(y)/y, g(z)/x]$

Please submit your solution until Monday, November 19, 2012 at 9:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution. Submission possibilities:

- By e-mail to `mbender@uni-koblenz.de` with the keyword "Homework DP" in the subject.

- Put it in the box in front of Room B 222.