Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Decision Procedures for Verification"
Exercise sheet 6

Exercise 6.1:
Prove or refute the following statement:

If t, s, s' are terms and x and y are distinct variables, then $(t[s/x])[s'/y] = t[s/x, s'/y]$.

Exercise 6.2:
Prove or refute the following statements:

(a) If F is a first-order formula, then F is valid if and only if $F \rightarrow \bot$ is unsatisfiable.

(b) If F and G are first-order formulae, F is valid, and $F \rightarrow G$ is valid, then G is valid.

(c) If F and G are first-order formulae, F is satisfiable, and $F \rightarrow G$ is satisfiable, then G is satisfiable.

(d) If F is a first-order formula and x a variable, then F is unsatisfiable if and only if $\exists x F$ is unsatisfiable.

(e) If F and G are first-order formulae and x is a variable then $\forall x(F \wedge G) \models \forall x F \wedge \forall x G$ and $\forall x F \wedge \forall x G \models \forall x(F \wedge G)$.

(f) If F and G are first-order formulae and x is a variable then $\exists x(F \wedge G) \models \exists x F \wedge \exists x G$ and $\exists x F \wedge \exists x G \models \exists x(F \wedge G)$.

Exercise 6.3:
Prove or refute the following statements:

(a) If F and G are first-order formulae and $F \models G$, then $F \models \neg G$ does not hold.

(b) If F and G are first-order formulae and $F \models G$, then $\neg F \models G$ does not hold.

(c) If F, G, and H are first-order formulae and $F \wedge G \models H$, then $F \models H$.

(d) If F, G, and H are first-order formulae and $F \vee G \models H$, then $F \models H$.

(e) If F and G are first-order formulae then if F and G are satisfiable then $F \wedge G$ is satisfiable.

Exercise 6.4:
Let $\Sigma = (\Omega, \Pi)$, where $\Omega = \{0/0, s/1, +/2\}$ and $\Pi = \emptyset$ (i.e. the only predicate symbol is $\approx$). Consider the following formulae in the signature Σ:

1. $F_1 = \forall x\ (x + 0 \approx x)$
2. $F_2 = \forall x, y\ (x + s(y) \approx s(x + y))$
3. $F_3 = \forall x, y\ \ ((x + y) + z \approx x + (y + z))$.

Find a Σ-structure in which F_1 and F_2 are valid but F_3 is not.

Supplementary exercise:

Exercise 6.5:
Let Σ be a signature, $\mathcal{A}$ a Σ-structure, $\beta : X \to U_{\mathcal{A}}$ a variable assignment and σ a substitution.

Prove:

(1) For any Σ-term t, $\mathcal{A}(\beta)(\sigma(t)) = \mathcal{A}(\beta \circ \sigma)(t)$
where $\beta \circ \sigma : X \to \mathcal{A}$ is the assignment $\beta \circ \sigma(x) = \mathcal{A}(\beta)(\sigma(x))$.

(2) For any Σ-formula F, $\mathcal{A}(\beta)(F\sigma) = \mathcal{A}(\beta \circ \sigma)(F)$.

Conclude that $\mathcal{A}, \beta \models F\sigma \quad \Leftrightarrow \quad \mathcal{A}, \beta \circ \sigma \models F$.

Hint: (1) can be proved by structural induction (induction on the structure of terms); (2) can be proved by induction on the structure of formulae using (1).

Please submit your solution until Monday, November 26, 2012 at 9:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution. Submission possibilities:

- By e-mail to `mbender@uni-koblenz.de` with the keyword "Homework DP" in the subject.
- Put it in the box in front of Room B 222.