Universität Koblenz-Landau

FB 4 Informatik

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November 26, 2012

Exercises for "Decision Procedures for Verification"
Exercise sheet 7

Exercise 7.1:
What is the clausal normal form of

$$\exists x\, \forall y\, (\forall z\, (p(y, z) \vee \neg\, x \approx y) \rightarrow (\forall z\, q(y, z) \wedge \neg\, r(x, y)))$$

Exercise 7.2:
Compute a most general unifier of

$$\{\, f(x, g(x)) = y,\ h(y) = h(v),\ v = f(g(z), w)\,\}$$

using the method presented in the lecture.

Exercise 7.3:
Let $\Sigma = (\Omega, \Pi)$ with $\Omega = \{b/0, f/1\}$ and $\Pi = \{p/1\}$.

(a) Which is the universe of the Herbrand interpretations over this signature?
If $\mathcal{A}$ is a Herbrand interpretation over Σ how are $b_{\mathcal{A}}$ and $f_{\mathcal{A}}$ defined?

(b) How many different Herbrand interpretations over Σ do exist? Explain briefly.

(c) How many different Herbrand models over Σ does the formula:

$$p(f(f(b))) \wedge \forall x(p(x) \rightarrow p(f(x))) \quad (1)$$

have? Explain briefly.

(d) Every Herbrand model over Σ of (1) is also a model of

$$\forall x p(f(f(x))) \quad (2)$$

Give an example of an algebra that is a model of (1) but not of (2).

Exercise 7.4:
Let $\Sigma = (\Omega, \Pi)$, where $\Omega = \{a/0, f/1, g/1\}$ and $\Pi = \{p/2\}$.

Use the resolution calculus Res described in the lecture to show that the following set of clauses (where x, y, z are variables) is unsatisfiable:

$$p(a, z)$$
$$\neg p(f(f(a)), a)$$
$$\neg p(x, g(y)) \vee p(f(x), y)$$

For computing the most general unifiers use the method presented in the lecture.

Exercise 7.5:
Consider the following formulae:

- $F_1 := \forall x(S(x) \rightarrow \exists y(R(x, y) \wedge P(y)))$
- $F_2 := \forall x(P(x) \rightarrow Q(x))$
- $F_3 := \exists x S(x)$
- $G := \exists x \exists y(R(x, y) \wedge Q(y))$

Use resolution to prove that $\{F_1, F_2, F_3\} \models G$.

Please submit your solution until Monday, December 3, 2012 at 9:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution. Submission possibilities:

- By e-mail to `mbender@uni-koblenz.de` with the keyword "Homework DP" in the subject.
- Put it in the box in front of Room B 222.