Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Decision Procedures for Verification"
Exercise sheet 8

Exercise 8.1:
Compute a most general unifier of

(1) $\{f(x, y) \doteq f(g(y), h(x, x))\}$

(2) $\{f(x, a) \doteq f(h(y, a), y), h(w, z) \doteq h(g(x), g(x))\}$

using the method presented in the lecture.

Exercise 8.2:
Let $\succ$ be a total and well-founded ordering on ground atoms such that, if the atom A contains more symbols than B, then $A \succ B$. Let N be the following set of clauses:

$$\begin{gathered}\neg q(z, z) \\ \neg q(f(x), y) \vee q(f(f(x)), y) \vee p(x) \\ \neg p(a) \vee \neg p(f(a)) \vee q(f(a), f(f(a))) \\ p(f(x)) \vee p(g(y)) \\ \neg p(g(a)) \vee p(f(f(a)))\end{gathered}$$

(a) Which literals are maximal in the clauses of N?
(b) Define a selection function S such that N is saturated under $Res_S^{\succ}$. Justify your choice.

Exercise 8.3:
Consider the notion of redundancy defined in the slides from December 3.

Assume that $S \succ P \succ Q \succ R$. Is the clause $C = \neg P \vee S$ redundant w.r.t. the set of clauses

$$N = \{\neg Q \vee \neg P, R \vee P, Q \vee S\}?$$

Justify your answer.

Exercise 8.4:
Which of the following formulae are in the Bernays-Schönfinkel class?

(1) $\exists y \forall x \ \ ((p(x) \vee r(x, y)) \wedge q(y))$
(2) $\forall x \exists y \forall z \exists u \ \ ((p(x) \vee q(y)) \wedge (q(y) \vee p(u)))$

(3) $\exists y \forall x \exists z \ \ ((r(x,y) \vee r(y,z)) \wedge q(z) \wedge r(y,z))$

(4) $\exists z \forall x \forall y \exists z' \ \ ((r(x,y) \vee r(y,z)) \wedge s(z,y,z'))$

Exercise 8.5:

Let $\Sigma = (\Omega, \Pi)$. Assume that Ω does not contain any function of arity greater than or equal to 1 (i.e. Ω consists only of constants).

Let $k \in \mathbb{N}$ and let H be a set of Horn clauses (in first-order logic without equality) in which every clause contains at most k variables.

Show that the satisfiability of H can be checked in polynomial time in the size of H.

Supplementary exercise

Exercise 8.6:

A set of Horn clauses in first-order logic without equality is called *superficial* if for every clause

$$A_1 \wedge \cdots \wedge A_n \rightarrow A \qquad \text{(alternatively written also:} \neg A_1 \vee \cdots \vee \neg A_n \vee A\text{)}$$

in H (where $A_1, \ldots, A_n, A$ are atoms), for every term t occurring in A there exists $j \in \{1, \ldots, n\}$ such that t occurs in A_j.

Let H be a superficial set of Horn clauses and let G be a conjunction of ground literals.

Prove that $H \cup G$ is satisfiable if and only if $H[G] \cup G$ is satisfiable,
where $H[G]$ is the set of all ground instances of H which contain only ground terms occurring in G or which already occur in H.

Please submit your solution until Monday, December 10, 2012 at 9:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution. Submission possibilities:

- By e-mail to `mbender@uni-koblenz.de` with the keyword "Homework DP" in the subject.
- Put it in the box in front of Room B 222.