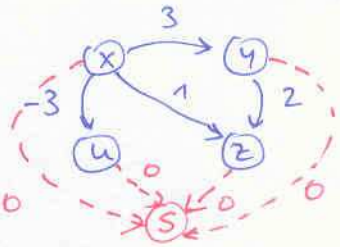
DIFFERENCE LOGIC : Examples

Check the satisfiability of the following formulae in (positive) difference logic (in $\mathbb{Q}$). In case of satisfiability find a satisfying assignment using the method presented in the lecture.

(1) $x-y \leq 3 \wedge y-z \leq 2 \wedge x-z \leq 1 \wedge x-u \leq -3$

Associated graph :



no negative cycles
$\Rightarrow$ formula is satisfiable.

To compute a satisfying assignment we add a new vertex $\underline{s}$ and edges with weight 0 from any node to $\underline{s}$

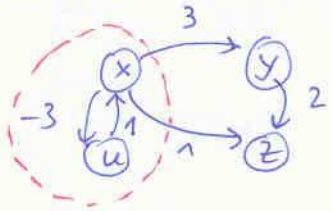
let δ_{xs} be the length of the shortest path from vertex $\underline{x}$ to $\underline{s}$.
δ_{ys} — " — — " — — " — — " — y
δ_{zs} — " — — " — — " — — " — z
δ_{us} — " — — " — — " — — " — u

Then $\beta : X \to \mathbb{Q}$ defined by $\beta(x) = \delta_{xs}$ for every $x \in X$ is a satisfying assignment.

$\delta_{xs} = -3$, $\delta_{ys} = 0$, $\delta_{zs} = 0$, $\delta_{us} = 0$ $\Rightarrow$ $\beta : X \to \mathbb{Q}$ defined by $\beta(x) = -3$, $\beta(y) = \beta(z) = \beta(u) = 0$ is a satisfying assignment.

(2) $x-y \leq 3 \wedge y-z \leq 2 \wedge x-z \leq 1 \wedge x-u \leq -3 \wedge u-x \leq 1$

Associated graph :



cycle of negative total weight

$\Rightarrow$ the formula is unsatisfiable.