

FOURIER-MOTZKIN: Example

$$F = \exists x \forall y \exists z (y \leq 0 \vee (-x \leq y+z \wedge z \leq x+y))$$

closed formula.

Task: Is F valid? Is F unsatisfiable?

Fact: In ODAG: F valid iff F equivalent to T
 F unsatisfiable iff F equivalent to \perp .

Solution In order to check the validity of F we successively eliminate the quantified variables.

1) Eliminate $\exists z$:

DNF

$$\exists x \forall y \exists z (y \leq 0 \vee (-x \leq y+z \wedge z \leq x+y))$$

$$\equiv \exists x \forall y (y \leq 0 \vee \exists z (-x \leq y+z \wedge z \leq x+y))$$

$$\equiv \exists x \forall y (y \leq 0 \vee \exists z (z \geq -x-y \wedge z \leq x+y))$$

$$\equiv \exists x \forall y (y \leq 0 \vee -x-y \leq x+y)$$

2) Eliminate $\forall y$

$$\exists x \forall y (y \leq 0 \vee -x-y \leq x+y) \equiv \exists x \exists y (y > 0 \wedge -x-y \leq x+y)$$

$$\equiv \exists x \exists y (y > 0 \wedge -x-y > x+y)$$

$$\equiv \exists x \exists y (y > 0 \wedge y < -x)$$

$$\equiv \exists x \exists y (0 < -x)$$

$$\equiv \exists x (0 < -x)$$

3) Eliminate $\exists x$

$$\exists x (x > 0) \equiv T$$

(only one lower bound.)

Comments:

All inequalities are written s.t. z is isolated on the left-hand side of inequality.

Combine upper/lower bounds & use transitivity.

Inequalities are written s.t. y on r.h. side.

Combine upper/lower bounds for y .

Conclusion

$F \equiv T$, therefore F is valid.