Universität Koblenz-Landau

FB 4 Informatik

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Collection of exercises: Part 1

Exercise 1. Assume $R \succ Q \succ P$. Let N_1 be the following set of clauses:

$$\begin{array}{ll} (C_1) & \neg R \vee \neg P \\ (C_2) & Q \vee P \\ (C_3) & \neg Q \\ (C_4) & R \vee \neg P \vee Q \end{array}$$

Use the ordered resolution calculus $\mathsf{Res}^{\succ}$ described in the lecture for checking the satisfiability of the set N_1 of clauses.

Exercise 2. Assume $P \succ Q \succ R \succ S$. Let N_2 be the following set of clauses:

$$\begin{array}{ll} (C_1) & \neg Q \vee P \\ (C_2) & R \vee \neg P \\ (C_3) & Q \vee \neg S \\ (C_4) & \neg Q \vee S \end{array}$$

(1) Define a selection function S such that this set of clauses is saturated w.r.t. the ordered resolution calculus with selection $\mathsf{Res}^{\succ}_S$. Justify your choice.

(2) Sort the clauses according to $\succ_C$.

(3) Construct a model of N_2 using the canonical construction presented in the lecture.

Exercise 3. Give the definition of redundancy of a clause w.r.t. a set N of clauses.

Assume $P \succ S \succ Q \succ R$.

(1) Is the clause $P \vee \neg S$ redundant w.r.t. the set of clauses $\{\neg Q \vee P, R \vee \neg P, Q \vee \neg S\}$?

(2) Is the clause $\neg Q \vee R$ redundant w.r.t. the set of clauses $\{\neg Q \vee P, R \vee \neg P, Q \vee \neg S\}$?

Justify your answers.

Exercise 4. Let $\Sigma = (\{f/1, g/1, h/1, a\}, \{p/2, q/1, r/2\})$. Let X be a set of variables, and assume that $\{x, y, z, u, v, w, s, t\} \subseteq X$.

Let $\succ$ an ordering on ground atoms with the property that for all ground terms $t_1, ..., t_{12}$, $\neg p(t_1, t_2) \succ p(t_3, t_4) \succ \neg q(t_5, t_6) \succ q(t_7, t_8) \succ \neg r(t_9, t_{10}) \succ r(t_{11}, t_{12})$.

Let N be the following set of clauses:

$$\begin{array}{ll} (1) & \neg r(f(x), y) \vee p(g(x), x) \\ (2) & \neg q(h(g(z))) \vee \neg p(z, u) \\ (3) & q(h(v)) \\ (4) & r(w, g(s)) \vee p(t, f(s)) \end{array}$$

Use the ordered resolution calculus $\mathsf{Res}^{\succ}$ described in the lecture for checking the satisfiability of the set N of clauses.

Exercise 5. Consider the following formulae over a signature containing function symbols $\Omega = \{c/0, f/1\}$ and predicate symbols $\Pi = \{P/1\}$:

- $F_1 := P(c)$
- $F_2 := \forall x(P(x) \rightarrow P(f(x)))$
- $F_3 := P(f(f(f(c))))$.

Use resolution to prove that $\{F_1, F_2\} \models F_3$.

Exercise 6.

(a) Give definitions for the following fragments of first-order logic:
  - The Bernays-Schönfinkel class;
  - The Ackermann class.
  - The monadic class.

(b) What is the idea in the proof of decidability for the Bernays-Schönfinkel class?

(c) To which of these classes do the following formulae belong (note that they can be in more than one, or in none of the classes above):

  (1) $\exists y \forall x \ \ ((p(x) \vee r(x, y)) \wedge q(y))$
  (2) $\forall x \exists y \forall z \exists u((p(x) \vee q(y)) \wedge (q(y) \vee p(u)))$
  (3) $\exists z \forall x \exists y(p(x) \vee q(y)) \wedge q(z)$
  (4) $\exists x \forall y(p(x) \vee r(y)) \wedge q(y)$
  (5) $\forall x \exists y \forall z \exists u((p(x) \vee r(x, y)) \wedge (q(y) \vee p(u)))$
  (6) $\exists z \forall x \exists y(p(x) \vee r(x, y)) \wedge q(z)$

Exercise 7. Check the satisfiability of the following formulae using the DAG version of the Congruence Closure algorithm presented in the class:

1. $f(a, b) \approx f(b, a) \wedge f(c, a) \not\approx f(b, c)$
2. $f(g(a)) \approx g(f(a)) \wedge f(g(f(b))) \approx a \wedge f(b) \approx a \wedge g(f(a)) \not\approx a$
3. $f(f(f(a))) \approx f(a) \wedge f(f(a)) \approx a \wedge f(a) \not\approx a$

Exercise 8.

(1a) Check the satisfiability over $\mathbb{Z}$ of the following set of constraints in positive difference logic using the method presented in the lecture. In case of satisfiability find a satisfiable assignment.

  (a) $x - y \leq 4 \wedge y - z \leq 2 \wedge x - z \leq 2 \wedge z - x \leq -3$
  (b) $x - y \leq 4 \wedge y - z \leq 0 \wedge x - z \leq 4 \wedge z - x \leq -3 \wedge x - u \leq -4$

(1b) Check the satisfiability over $\mathbb{Z}$ of the following set of constraints in difference logic using the method presented in the lecture. In case of satisfiability find a satisfiable assignment.

(a) $x - y \leq 4 \wedge y - z \leq 0 \wedge x - z < 4 \wedge z - x \leq -3 \wedge x - u \leq -4$

(a) $x - y \leq 4 \wedge y - z \leq 0 \wedge x - z < 4 \wedge z - x < -3 \wedge x - u \leq -4$

(2a) Check the satisfiability over $\mathbb{Q}$ of the following sets of constraints in positive difference logic. In case of satisfiability find a satisfiable assignment.

(a) $x - y \leq 5 \wedge y - u \leq 4 \wedge x - z \leq -1 \wedge z - x \leq 1$.

(b) $x - y \leq 5 \wedge y - u \leq 4 \wedge x - z \leq -1 \wedge z - x \leq 1 \wedge z - y \leq -5$.

(2a) Check the satisfiability over $\mathbb{Q}$ of the following sets of constraints in difference logic. In case of satisfiability find a satisfiable assignment.

(a) $x - y \leq 5 \wedge y - u \leq 4 \wedge x - z < -1 \wedge z - x \leq 1$.

(b) $x - y \leq 5 \wedge y - u \leq 4 \wedge x - z < -0.5 \wedge z - x < 1 \wedge z - y \leq -5$.