

Exercises for “Decision Procedures for Verification” Exercise sheet 12

In what follows we consider the theory of arrays defined in the lecture. We assume that the theory of indices \mathcal{T}_i is $LI(\mathbb{Z})$, and the theory of elements \mathcal{T}_e is $LI(\mathbb{Q})$.

Exercise 12.1: (2 P)

Which of the formulae below are (equivalent to formulae) in the array property fragment and which are not?

Justify your answer. (The universally quantified variables i, j are sort index; the indices k, l which are not universally quantified are considered to be constants of sort index)

- (1) $\forall i (a[i + 1] > a[i])$
- (2) $\forall i (i < a[k] \rightarrow a[i] = a[k])$
- (3) $\forall i, j (l_1 \leq i \leq u_1 < l_2 \leq j \leq u_2 \rightarrow a[i] \leq a[j])$
- (3) $\forall i, j (l_1 < i \leq u_1 < l_2 \leq j \leq u_2 \rightarrow a[i] \leq a[j]).$

Exercise 12.2: (4 P)

Consider the array property formula:

$$F : \text{write}(a, l, v)[k] = b[k] \wedge b[k] \neq v \wedge a[k] = v \wedge \forall i (i \neq l \rightarrow a[i] = b[i])$$

and let F'_6 be the formula obtained (in the example presented in the lecture) by applying Steps 1–6 to F , after simplification.

$$\begin{aligned} F'_6 : \quad & a'[k] = b[k] \wedge b[k] \neq v \wedge a[k] = v \wedge (k \neq l \rightarrow a[k] = b[k]) \\ & \wedge a[\lambda] = b[\lambda] \wedge a[l - 1] = b[l - 1] \wedge a[l + 1] = b[l + 1] \\ & \wedge a'[l] = v \wedge (k \neq l \rightarrow a[k] = a'[k]) \\ & \wedge a[\lambda] = a'[\lambda] \wedge a[l - 1] = a'[l - 1] \wedge a[l + 1] = a'[l + 1] \\ & \wedge \lambda \neq k \wedge \lambda \neq l \wedge \lambda \neq l - 1 \wedge \lambda \neq l + 1. \end{aligned}$$

Check the satisfiability of F'_6 w.r.t. $\mathcal{T} = UIF_{\{a,b,a'\}} \cup \mathcal{T}_i \cup \mathcal{T}_e$ using one of the versions of the $DPLL(\mathcal{T})$ procedure presented in the class. For theory reasoning in \mathcal{T} use the Nelson-Oppen procedure.

Exercise 12.3: (4 P)

Consider the following array property formula:

$$F : \forall i (l \leq i \leq u \rightarrow a[i] = b[i]) \wedge \neg \forall i (l \leq i \leq u + 1 \rightarrow \text{write}(a, u + 1, b[u + 1])[i] = b[i])$$

Apply to the formula F the Steps 1–6 of the transformation procedure for formulae in the array property fragment presented in the lecture.

Please submit your solution until Monday, January 27, 2014 at 16:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to sofronie@uni-koblenz.de with the keyword “Homework DP” in the subject.
- Put it in the box in front of Room B 222.