

Exercises for “Decision Procedures for Verification” Exercise sheet 2

Exercise 2.1: (2 P)

Prove Prop. 1.4: If N is a set of propositional formulas, then $N \models F$ if and only if $N \cup \{\neg F\}$ is unsatisfiable.

(A set of propositional formulas is unsatisfiable, if and only if for every valuation \mathcal{A} there is a formula G in the set such that $\mathcal{A} \not\models G$.)

Exercise 2.2: (2 P)

Let F be a formula, P a propositional variable not occurring in F , and F' a subformula of F . Prove: The formula $F[P] \wedge (P \leftrightarrow F')$ is satisfiable if and only if $F[F']$ is satisfiable.

Exercise 2.3: (4 P)

Let F be the following formula:

$$\neg[((Q \wedge \neg P) \wedge \neg(Q \wedge R)) \rightarrow (Q \wedge (Q \rightarrow P) \wedge \neg P)] \wedge (P \vee R)$$

- (1) Compute the negation normal form (NNF) F' of F .
- (2) Convert F' to CNF using the satisfiability-preserving transformation described in the lecture.

Exercise 2.4: (2 P)

Determine the polarity of the following subformulae of

$$F = \neg((\neg(P \wedge \neg Q)) \vee (R \vee \neg S)) \vee (U \wedge V)$$

- (1) $(P \wedge \neg Q)$
- (2) Q
- (3) $(R \vee \neg S)$
- (4) S
- (5) V
- (6) $((\neg(P \wedge \neg Q)) \vee (R \vee \neg S))$

Supplementary exercises

Exercise 2.5: (2 P)

Consider the formulae $F_n = \bigvee_{i=1}^n (Q_i \wedge R_i)$ for $n \in \mathbb{N}$.

As a function of n , how many clauses are in:

- (1) the CNF formula F' constructed using the distributivity of disjunctions over conjunctions?
- (2) the CNF formula F'' obtained using the satisfiability-preserving translation to clause form?
- (3) For which n is the first approach better?

Exercise 2.6: (4 P)

Let F be a formula containing neither \rightarrow nor \leftrightarrow , P a propositional variable not occurring in F , and F' a subformula of F . Prove:

- If F' has positive polarity in F then $F[F']$ is satisfiable if and only if $F[P] \wedge (P \rightarrow F')$ is satisfiable.
- If F' has negative polarity in F then $F[F']$ is satisfiable if and only if $F[P] \wedge (F' \rightarrow P)$ is satisfiable.

Reminder: The structural induction principle (for propositional logic).

Let \mathcal{B} be a property of formulae in propositional logic. Assume that the following hold:

- for every propositional variable $P \in \Pi$, P has property \mathcal{B} ;
- \perp and \top have property \mathcal{B} ;
- if $F = F_1 \text{ op } F_2$ for $\text{op} \in \{\vee, \wedge, \rightarrow, \leftrightarrow\}$ and if both F_1 and F_2 have property \mathcal{B} then F has property \mathcal{B} ;
- if $F = \neg F_1$ and F_1 has property \mathcal{B} then F has property \mathcal{B} .

Then property \mathcal{B} holds for all Π -formulae.

Please submit your solution until Wednesday, November 6, 2013 at 14:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to sofronie@uni-koblenz.de with the keyword “Homework DP” in the subject.
- Put it in the box in front of Room B 222.