Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans **November 5, 2013**

Exercises for "Decision Procedures for Verification"
Exercise sheet 3

Exercise 3.1: (*3 P*)
Use the resolution calculus to prove that the following set of clauses is unsatisfiable:

(1)	$\neg P \vee \neg Q \vee R$
(2)	$\neg P \vee \neg Q \vee S$
(3)	P
(4)	$\neg S \vee \neg R$
(5)	Q

Exercise 3.2: (*1 P*)
Find a total ordering on the propositional variables A, B, C, D, E, such that the associated clause ordering $\succ_C$ orders the clauses like this:

$$B \vee C \succ_C A \vee A \vee \neg C \succ_C C \vee E \succ_C C \vee D \succ_C \neg A \vee D \succ_C \neg E.$$

Exercise 3.3: (*4 P*)
Let N be the following set of clauses:

(1)	$\neg P_3 \vee P_1 \vee P_1$
(2)	$\neg P_2 \vee P_1$
(3)	$P_4 \vee P_4$
(4)	P_4
(5)	$P_3 \vee \neg P_2$
(6)	$P_4 \vee P_3$

(1) Let $\succ$ be the ordering on propositional variables defined by $P_4 \succ P_3 \succ P_2 \succ P_1$. Sort the clauses in N according to $\succ_C$. Which literals are maximal in the clauses of N?
(2) Define a selection function S such that N is saturated under $Res_S^{\succ}$.
(3) Construct a model of N using the canonical construction presented in the lecture.

Exercise 3.4: (*2 P*)
Use a DPLL procedure to find a model of each of the following formulae, or prove that the particular formula has no model:

(1) $(P \vee \neg Q) \wedge (\neg P \vee Q) \wedge (Q \vee \neg R) \wedge (\neg Q \vee \neg R)$
(2) $(P \vee Q \vee \neg R) \wedge (P \vee \neg Q) \wedge (P \vee Q \vee R) \wedge (R \vee Q) \wedge (R \vee \neg Q) \wedge (\neg P \vee \neg R) \wedge \neg U$

Exercise 3.5: $(L\ P)$

et $\Sigma = (\Omega, \Pi)$ be a signature, where $\Omega = \{f/3,\ g/1,\ a/0,\ b/0\}$ and $\Pi = \{p/2\}$; let X be the set of variables $\{x, y, z\}$. Which of the following expressions are terms over Σ and X, which are atoms/literals/clauses/formulae (in first-order logic with equality), which are neither?

(a) $\neg p(g(a), f(x, y, g(a)))$

(b) $f(x, x, x) \approx x$

(c) $p(f(x, x, a), x) \vee p(a, b)$

(d) $p(\neg g(x), g(y))$

(e) $\neg p(f(x, y, y))$

(f) $\neg p(f(x, y), y) \vee p(x, y)$

(g) $p(a, b) \wedge p(x, y) \wedge y$

(h) $\exists y(\neg p(f(y, y, y), y))$

(i) $\forall x \forall y(f(p(x, y), x, x) \approx g(x))$

Additional Exercise

Let F be the following formula:

$$(\neg((Q \wedge \neg P) \wedge (\neg Q \vee \neg R)) \vee (Q \wedge (\neg Q \vee P))) \wedge (P \vee R).$$

Use the satisfiability preserving CNF translation (possibly optimized using the polarity of the subformulae) to compute a set N_F of clauses such that F is satisfiable if and only if N_F is satisfiable.

Please submit your solution until Monday, November 11, 2013 at 14:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework DP" in the subject.
- Put it in the box in front of Room B 222.