Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans

November 14, 2013

Exercises for "Decision Procedures for Verification"
Exercise sheet 4

Exercise 4.1: (*2 P*)
Assume $P \succ Q \succ R$. Let N be the following set of clauses:

(1)	$\neg R \vee P$
(2)	$\neg Q \vee \neg P$
(3)	Q
(4)	$R \vee P$

Let S be the selection function which selects $\neg R$ in clause (1) and $\neg Q$ in clause (2).

Use the ordered resolution calculus with selection $\mathsf{Res}_S^{\succ}$ described in the lecture for checking the satisfiability of the set N of clauses.

Exercise 4.2: (*2 P*)
Assume $S \succ P \succ Q \succ R$. Let N be the following set of clauses:

(1)	$\neg Q \vee \neg P$
(2)	$R \vee P$
(3)	$Q \vee S$
(4)	$\neg Q \vee \neg S$

(a) Define a selection function S such that this set of clauses is saturated w.r.t. the ordered resolution calculus with selection $\mathsf{Res}_S^{\succ}$. Justify your choice.

(b) Sort the clauses according to $\succ_C$.

(c) Construct a model of N using the canonical construction presented in the lecture.

Exercise 4.3: (*2 P*)
Compute the results of the following substitutions:

(a)	$f(g(x), x)[g(a)/x]$	(c)	$\forall y(p(f(y, x), x))[y/x]$
(b)	$p(f(y, x), g(x))[x/y]$	(d)	$\forall y(p(f(z, g(y)), g(x)) \vee \exists z(g(z) \approx y))[g(b)/z]$
(c)	$\forall y(p(f(y, x), g(y)))[x/y]$	(e)	$\exists y\big(f(x, y) \approx x \rightarrow \forall x(f(x, y) \approx x)\big)[g(y)/y, g(z)/x]$

Exercise 4.4: (*3 P*)
Reminder: A formula F is valid in a Σ-algebra (Σ-structure) $\mathcal{A}$ under assignment β (Nota-

tion: $\mathcal{A}, \beta \models F$) if $\mathcal{A}(\beta)(F) = 1$. F is valid in $\mathcal{A}$ (Notation: $\mathcal{A} \models F$) iff $\mathcal{A}, \beta \models F$, for all $\beta \in X \to U_{\mathcal{A}}$.

Let $\Sigma = \{0, s, +\}$. Consider the following formulae in the signature Σ:

1. $F_1 = \forall x \ (x + 0 \approx x)$
2. $F_2 = \forall x, y \ (x + s(y) \approx s(x + y))$
3. $F_3 = \forall x, y \ \ (x + y \approx y + x)$.

Find a Σ-structure in which F_1 and F_2 are valid but F_3 is not.

Exercise 4.5: (*2 P*)
What is the clausal normal form of

$$\exists x \, \forall y \, (\forall z \, (p(y, z) \vee \neg \, x \approx y) \to (\forall z \, q(y, z) \wedge \neg \, r(x, y)))$$

Supplementary exercise (will be discussed in the exercise session)

Exercise 4.6: (*5 P*)
Let H be a set of propositional Horn clauses. The size of H is the number of all literals which occur in H.

Prove that the resolution calculus $\mathsf{Res}_S^{\succ}$ (or the marking algorithm discussed in the lecture "Logik für Informatiker") can check the satisfiability of H in time polynomial in the size of H.

Can you give an algorithm for check the satisfiability of H in time linear in the size of H?

Please submit your solution until Monday, November 18, 2013 at 16:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework DP" in the subject.
- Put it in the box in front of Room B 222.