Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans **November 21, 2013**

Exercises for "Decision Procedures for Verification"
Exercise sheet 5

Exercise 5.1: *(4 P)*
Let $\Sigma = (\Omega, \Pi)$ with $\Omega = \{b/0, f/1\}$ and $\Pi = \{p/1\}$.

(a) Which is the universe of the Herbrand interpretations over this signature?
If $\mathcal{A}$ is a Herbrand interpretation over Σ how are $b_{\mathcal{A}}$ and $f_{\mathcal{A}}$ defined?

(b) How many different Herbrand interpretations over Σ do exist? Explain briefly.

(c) How many different Herbrand models over Σ does the formula:

$$p(f(f(b))) \wedge \forall x(p(x) \rightarrow p(f(x))) \quad (1)$$

have? Explain briefly.

(d) Every Herbrand model over Σ of (1) is also a model of

$$\forall x p(f(f(x))) \quad (2)$$

Give an example of an algebra that is a model of (1) but not of (2).

Exercise 5.2: *(2 P)*
Compute a most general unifier of

$$\{\, f(x, g(x)) = y,\ h(y) = h(v),\ v = f(g(z), w)\,\}$$

using the method presented in the lecture (cf. slides from 21.11.2013, page 42).

Exercise 5.3: *(2 P)*
Let $\Sigma = (\Omega, \Pi)$, where $\Omega = \{a/0, f/1, g/1\}$ and $\Pi = \{p/2\}$.

Use the resolution calculus Res (described on page 40 on the slides from 21.11.2011) to show that the following set of clauses (where x, y, z are variables) is unsatisfiable:

$$p(a, z)$$
$$\neg p(f(f(a)), a)$$
$$\neg p(x, g(y)) \vee p(f(x), y)$$

For computing the most general unifiers use the method presented in the lecture.

Exercise 5.4: *(3 P)*
Consider the following formulae:

- $F_1 := \forall x(S(x) \rightarrow \exists y(R(x,y) \wedge P(y)))$
- $F_2 := \forall x(P(x) \rightarrow Q(x))$
- $F_3 := \exists x S(x)$
- $G := \exists x \exists y(R(x,y) \wedge Q(y))$

Use resolution to prove that $\{F_1, F_2, F_3\} \models G$.

Exercise 5.5: *(3 P)*
Let $\succ$ be a total and well-founded ordering on ground atoms such that, if the atom A contains more symbols than B, then $A \succ B$. Let N be the following set of clauses:

$$\neg q(z,z)$$
$$\neg q(f(x),y) \vee q(f(f(x)),y) \vee p(x)$$
$$\neg p(a) \vee \neg p(f(a)) \vee q(f(a),f(f(a)))$$
$$p(f(x)) \vee p(g(y))$$
$$\neg p(g(a)) \vee p(f(f(a)))$$

(a) Which literals are maximal in the clauses of N?
(b) Define a selection function S such that N is saturated under $Res_S^{\succ}$. Justify your choice.

Please submit your solution until Monday, November 25, 2013 at 16:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword “Homework DP” in the subject.
- Put it in the box in front of Room B 222.