Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Decision Procedures for Verification"
Exercise sheet 6

Exercise 6.1: *(2 P)*

(1) Redundant clauses remain redundant, if the theorem prover derives new clauses and adds them to the current set of clauses. Prove:

If N and M are sets of clauses and $N \subseteq M$, then $\text{Red}(N) \subseteq \text{Red}(M)$.

(2) Redundant clauses remain redundant, if the theorem prover deletes redundant clauses. Prove:

If N and M are sets of clauses and $M \subseteq \text{Red}(N)$, then $\text{Red}(N) \subseteq \text{Red}(N \backslash M)$.

Exercise 6.2: *(2 P)*
Assume $S \succ P \succ Q \succ R$. Let N be the following set of clauses:

$$\begin{array}{ll} (1) & \neg Q \vee \neg P \\ (2) & R \vee P \\ (3) & Q \vee S \\ (4) & \neg Q \vee \neg S \end{array}$$

Give the definition of redundancy of a clause w.r.t. a set of clauses. Is the clause $\neg P \vee S$ redundant w.r.t. the set N above? Justify your answer.

Exercise 6.3: *(2 P)*
To which of the classes discussed in the lecture (the Bernays-Schönfinkel class, the Ackermann class or the monadic class) do the following formulae belong:

(1) $\exists y \forall x \ \ ((p(x) \vee r(x, y)) \wedge q(y))$

(2) $\forall x \exists y \forall z \exists u ((p(x) \vee q(y)) \wedge (q(y) \vee p(u))$

(3) $\exists z \forall x \exists y (p(x) \vee q(y)) \wedge q(z)$

(4) $\exists x \forall y (P(x) \vee R(y)) \wedge Q(y)$

Note that they can be in more than one, or in none of the classes above.

Exercise 6.4: *(7 P)*
Let $\Sigma = (\{c_1/0, \dots, c_n/0, f_1/1, \dots, f_n/1\}, \Pi)$ be a signature. Consider the following classes of clauses:

- G (denoted in the lecture also $G(c_1, \dots, c_n)$) is the class of all ground clauses in the signature Σ which do not contain any occurrence of a unary function symbol.
- V (denoted in the lecture also $V(x, c_1, \dots, c_n)$) is the class of all clauses with one variable (x) in the signature Σ which do not contain any occurrence of a unary function symbol.
- G_f (denoted in the lecture also $G(c_1, \dots, c_n, f_k(c_j))$) is the class of all ground clauses in the signature Σ which contain at least one occurrence of a unary function symbol (having as argument a constant); no nested applications of unary function symbols are allowed.

  Example: Assume $p/3.q/2 \in \Pi$ Then:
  $C_1 : p(c_1, c_2, c_3) \vee \neg q(c_2, c_1) \notin G_f$
  $C_2 : q(c_1, c_2) \vee \neg q(f_1(c_3), c_4) \in G_f$
  $C_3 : q(c_1, c_2) \vee \neg q(f_1(c_3), f_2(f_3(c_4))) \notin G_f$.

- V_f (denoted in the lecture also $V(x, c_1, \dots, c_n, f_j(x))$) is the class of all ground clauses in the signature Σ which contain only one variable (x), at least one occurrence of a unary function symbol (having as argument the variable x), no occurrences of terms of the form $f_k(c_j)$; in addition no nested applications of unary function symbols are allowed.

  Example: Assume $p/3.q/2 \in \Pi$ Then:
  $C'_1 : p(x, c_2, x) \vee \neg q(c_2, c_1) \notin G_f$
  $C'_2 : q(x, c_2) \vee \neg q(f_1(c_3), x) \notin G_f$
  $C'_3 : q(c_1, x) \vee \neg q(x, f_2(f_3(x))) \notin G_f$ $C'_4 : p(x, c_2, x) \vee \neg p(c_2, x, f(x)) \in G_f$.

Consider a term ordering $\succ$ in which $f(t) \succ t$ for every term t and terms containing function symbols of arity 1 are larger than those who do not. Consider the general ordered resolution calculus $\mathsf{Res}^{\succ}$. Prove that in this calculus:

(1) The resolvent of a clause in G_f and a clause in V_f is a clause in G_f of G.

(2) The resolvent of two clauses in V_f is a clause in G, G_f, V or V_f.

Please submit your solution until Monday, December 2, 2013 at 16:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework DP" in the subject.
- Put it in the box in front of Room B 222.