Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Decision Procedures for Verification"
Exercise sheet 9

Exercise 9.1: *(3 P)*
Check the satisfiability of the following formulae in difference logic w.r.t. $\mathbb{Z}$; in case of satisfiability find a satisfying assignment.

(1) $\phi_1 = x - y < 4 \ \wedge \ y - z \leq 2 \ \wedge \ z - x < -3 \ \wedge \ x - u \leq -3.$

(2) $\phi_2 = x - y < 4 \ \wedge \ y - z \leq 2 \ \wedge \ z - x \leq -5 \ \wedge \ x - u < -3 \ \wedge u - x \leq 4.$

(3) $\phi_3 = x - y < 4 \ \wedge \ y - z \leq 2 \ \wedge \ z - x < -5 \ \wedge \ x - u < -3 \ \wedge u - x \leq 4.$

Hint: It is sufficient to check the existence of negative cycles in $G(\phi_i)$ by looking at the graphs; in this assignment you do not have to use the Bellman-Ford algorithm for this.

Exercise 9.2: *(3 P)*
Check the satisfiability of the following formulae in difference logic w.r.t. $\mathbb{Q}$; in case of satisfiability find a satisfying assignment.

(1) $\phi_1 = x - y < 4 \ \wedge \ y - z \leq 2 \ \wedge \ z - x < -5 \ \wedge \ x - u \leq -3.$

(2) $\phi_2 = x - y < 4 \ \wedge \ y - z \leq 2 \ \wedge \ z - x \leq -6 \ \wedge \ x - u \leq -4 \ \wedge u - x \leq 4.$

(3) $\phi_3 = x - y < 4 \ \wedge \ y - z \leq 2 \ \wedge \ z - x \leq -7 \ \wedge \ x - u < -3 \ \wedge u - x \leq 4.$

Hint: It is sufficient to check the existence of negative cycles in $G(\phi_i)$ by looking at the graphs; in this assignment you do not have to use the Bellman-Ford algorithm for this.

Exercise 9.3: *(4 P)*
Let F be the following conjunction (in linear rational arithmetic $LI(\mathbb{Q})$):

$$
\begin{array}{lrcll}
F: & x_1 + x_2 + 2x_3 & = & 2 & \wedge \\
 & x_1 + x_3 + \frac{1}{5} & < & 0 & \wedge \\
 & x_2 - x_3 & \leq & \frac{1}{2} & \wedge \\
 & x_1 + 5x_3 & \leq & 5 &
\end{array}
$$

Check the satisfiability of F using:

(1) the Fourier-Motzking method for quantifier elimination;

(2) the Loos-Weispfenning method for quantifier elimination.

Exercise 9.4: (*2 P*)
Consider the following formulae (in linear rational arithmetic $LI(\mathbb{Q})$):

$$F_1 = \exists x \forall y \exists z (y > 0 \vee (x + y - z < 0 \wedge x + y + z < 0))$$
$$F_2 = \forall x \exists y \exists z (2x - y > 0 \wedge 2y - z > 0 \wedge z - y \geq 2 \wedge x - y < 0 \wedge y < 0)$$

Check whether F_1 and F_2 are valid or satisfiable using the Fourier-Motzkin method for quantifier elimination.

Please submit your solution until Monday, January 13, 2014 at 16:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword “Homework DP” in the subject.
- Put it in the box in front of Room B 222.