

Exercises for “Decision Procedures for Verification”  
Exercise sheet 1

**Exercise 1.1:** (5 P)

Determine which of the following formulas are valid/satisfiable/unsatisfiable:

- (1)  $(P \wedge Q) \rightarrow (P \vee Q)$
- (2)  $(P \vee Q) \rightarrow (P \wedge Q)$
- (3)  $\neg(P \wedge \neg\neg P)$
- (4)  $Q \rightarrow \neg Q$
- (5)  $Q \wedge \neg Q$
- (6)  $\neg(\neg P \vee \neg\neg P)$
- (7)  $((P \rightarrow Q) \wedge (\neg P \rightarrow R)) \rightarrow (Q \vee R)$

**Exercise 1.2:** (5 P)

If  $F$  and  $G$  are propositional formulae then prove that the following are equivalent:

- (a)  $F \models G$ ;
- (b)  $\models F \rightarrow G$  (i.e.  $F \rightarrow G$  is valid);
- (c)  $F \wedge \neg G$  is unsatisfiable.

**Exercise 1.3:** (2 P)

Prove: If  $N$  is a set of propositional formulas, then  $N \models F$  if and only if  $N \cup \{\neg F\}$  is unsatisfiable.

(A set of propositional formulas is unsatisfiable, if and only if for every valuation  $\mathcal{A}$  there is a formula  $G$  in the set such that  $\mathcal{A} \not\models G$ .)

**Exercise 1.4:** (5 P)

Prove:

- (1) If  $F_1, \dots, F_n, G$  are propositional formulae then  $F_1 \wedge \dots \wedge F_n \rightarrow G$  is valid iff every valuation which is a model of all the formulae  $F_1, F_2, \dots, F_n$  is also a model of  $G$ .

(2) If  $F_1, \dots, F_n, G$  are propositional formulae then the following are equivalent:

- (a)  $\{F_1, \dots, F_n\} \models G$
- (b)  $F_1 \wedge \dots \wedge F_n \rightarrow G$  is valid
- (c)  $F_1 \wedge \dots \wedge F_n \wedge \neg G$  is unsatisfiable.

### Supplementary exercises

#### Exercise 1.5: (2 P)

Consider the formulae  $F_n = \bigvee_{i=1}^n (Q_i \wedge R_i)$  for  $n \in \mathbb{N}$ .

As a function of  $n$ , how many clauses are in:

- (1) the CNF formula  $F'$  constructed using the distributivity of disjunctions over conjunctions?
- (2) the CNF formula  $F''$  obtained using the satisfiability-preserving translation to clause form?
- (3) For which  $n$  is the first approach better?

*You will be able to solve 1.5(2) and (3) only after the satisfiability-preserving translation to clauses normal form is presented in the lecture on Tuesday, 4.11.2014.*

Please submit your solution until Wednesday, November 5, 2014 at 16:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to [sofronie@uni-koblenz.de](mailto:sofronie@uni-koblenz.de) with the keyword “Homework DP” in the subject.
- Put it in the box in front of Room B 222.