Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans **January 27, 2015**

Exercises for "Decision Procedures for Verification"
Exercise sheet 11

Exercise 11.1: (*2 P*)
Let $\mathcal{T}$ be the combination of $LI(\mathbb{Q})$ (linear arithmetic over $\mathbb{Q}$) and UIF_Σ, the theory of uninterpreted function symbols in the signature $\Sigma = \{\{f/1, g/2\}, \emptyset\}$.
Check the satisfiability of the following ground formula w.r.t. $\mathcal{T}$ using the deterministic version of the Nelson-Oppen procedure (after purifying the formulae check for entailment of equalities between shared constants and propagate the entailed equalities):

- $\phi = (g(c+d, e) \approx f(g(c, d)) \wedge c + e \approx d \wedge e \geq 0 \wedge c \geq d \wedge g(c, c) \approx e \wedge f(e) \not\approx g(c+c, 0))$

Exercise 11.2: (*4 P*)
Use the Nelson-Oppen procedure for checking the satisfiability of the following formulae:

1. $1 \leq c \wedge c \leq 3 \wedge f(c) \not\approx f(1) \wedge f(c) \not\approx f(3) \wedge f(1) \not\approx f(2)$
   in the combination $LI(\mathbb{Z}) \cup UIF_{\{f\}}$.
2. $f(c) \approx f(c+d) \wedge 1 \leq c \wedge c \leq d + e \wedge c + e \leq d \wedge d = 1 \wedge f(c) \not\approx f(2)$
   in the combination $LI(\mathbb{Z}) \cup UIF_{\{f\}}$.

Exercise 11.3: (*2 P*)
Check the satisfiability w.r.t. $\mathcal{T} = LI(\mathbb{Q})$ of the following set of ground clauses using the "lazy" approach to SMT presented in the class.

$$(\neg(0 \leq x) \vee \neg(y \leq z)) \;\wedge\; (\neg(z \leq x+y) \vee (y \leq z)) \;\wedge\; (\neg(0 \leq y) \vee (0 \leq x)) \;\wedge\; (z \leq x+y)$$

For theory reasoning in $LI(\mathbb{Q})$ use the Fourier-Motzkin algorithm.

Exercise 11.4: (*2 P*)
Let $\mathcal{T} = LI(\mathbb{Q})$, and let $Q := x \geq 1, R := x \leq y, P := x + x \leq 2$. Use a DPLL($\mathcal{T}$) method to check the satisfiability w.r.t. $\mathcal{T}$ of the following set of clauses:

$$\begin{array}{ll} (C_1) & \neg R \vee P \\ (C_2) & \neg Q \vee \neg P \\ (C_4) & R \vee P \end{array}$$

Supplementary exercises:

Exercise 11.5: (*5 P*)
Let $\mathcal{T}$ be a theory with signature Σ and $\mathsf{Mod}(\mathcal{T})$ be its class of models.
Show that if $\mathsf{Mod}(\mathcal{T})$ is closed under products then $\mathcal{T}$ is convex.

Exercise 11.6: (*5 P*)
We say that a theory $\mathcal{T}$ is *stably infinite* if for every quantifier-free formula ϕ, ϕ is satisfiable in $\mathcal{T}$ iff ϕ is satisfiable in a (countably) infinite model of $\mathcal{T}$.
Let $\mathcal{T}_1, \mathcal{T}_2$ be stably infinite theories with disjoint signatures. Prove that their combination $\mathcal{T}_1 \cup \mathcal{T}_2$ is stably infinite.

Please submit your solution until Wednesday, February 4, 2015 at 13:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework DP" in the subject.
- Put it in the box in front of Room B 222.