

### Exercises for “Decision Procedures for Verification” Exercise sheet 3

#### Exercise 3.1: (2 P)

Assume  $S \succ P \succ Q \succ R$ . Let  $N$  be the following set of clauses:

- (1)  $\neg Q \vee \neg P$
- (2)  $R \vee P$
- (3)  $Q \vee S$
- (4)  $\neg Q \vee \neg S$

- (1) Which literals are maximal in the clauses of  $N$ ?
- (2) Which inferences are possible in the ordered resolution calculus  $\text{Res}^{\succ}$  with the rules:

$$\frac{C \vee A \quad D \vee \neg A}{C \vee D} \quad [\text{ordered resolution}]$$

if  $C, D$  are clauses and  $A$  is a propositional variable with:

- (i)  $A \succ C$  ( $A$  is larger (in  $\succ$ ) than the maximal literal in  $C$ );
- (ii)  $\neg A \succeq \max(D)$  (i.e.  $\neg A$  is larger than or equal to the maximal literal of  $D$ ).

$$\frac{C \vee A \vee A}{(C \vee A)} \quad [\text{ordered factoring}]$$

if  $C$  is a clause and  $A$  a propositional variable such that  $A$  is maximal in  $C$ .

- Let  $S$  be the selection function which selects the negative literal  $\neg Q$  in the clauses (1) and (4). Which inferences are possible in the ordered resolution calculus with selection  $\text{Res}_S^{\succ}$  presented in the lecture.

#### Exercise 3.2: (1 P)

Find a total ordering on the propositional variables  $A, B, C, D, E$ , such that the associated clause ordering  $\succ_C$  orders the clauses like this:

$$B \vee C \succ_C A \vee A \vee \neg C \succ_C C \vee E \succ_C C \vee D \succ_C \neg A \vee D \succ_C \neg E.$$

**Exercise 3.3:** (4 P)

Let  $N$  be the following set of clauses:

- (1)  $\neg P_3 \vee P_1 \vee P_1$
- (2)  $\neg P_2 \vee P_1$
- (3)  $P_4 \vee P_4$
- (4)  $P_4$
- (5)  $P_3 \vee \neg P_2$
- (6)  $P_4 \vee P_3$

- (1) Let  $\succ$  be the ordering on propositional variables defined by  $P_4 \succ P_3 \succ P_2 \succ P_1$ . Sort the clauses in  $N$  according to  $\succ_C$ . Which literals are maximal in the clauses of  $N$ ?
- (2) Define a selection function  $S$  such that  $N$  is saturated under  $Res_S^\succ$ .
- (3) Construct a model of  $N$  using the canonical construction presented in the lecture.

**Exercise 3.4:** (2 P)

Use a DPLL procedure to find a model of each of the following formulae, or prove that the particular formula has no model:

- (1)  $(P \vee \neg Q) \wedge (\neg P \vee Q) \wedge (Q \vee \neg R) \wedge (\neg Q \vee \neg R)$
- (2)  $(P \vee Q \vee \neg R) \wedge (P \vee \neg Q) \wedge (P \vee Q \vee R) \wedge (R \vee Q) \wedge (R \vee \neg Q) \wedge (\neg P \vee \neg R) \wedge \neg U$

Please submit your solution until Wednesday, November 19, 2014 at 13:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to [sofronie@uni-koblenz.de](mailto:sofronie@uni-koblenz.de) with the keyword “Homework DP” in the subject.
- Put it in the box in front of Room B 222.