Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Decision Procedures for Verification"
Exercise sheet 9

Exercise 9.1: (*2 P*)
Check the satisfiability of the following ground formula using the algorithm based on congruence closure presented in the lecture.

- $\phi = h(c, e) \approx d \wedge g(d) \approx e \wedge g(h(c, g(d))) \not\approx e.$

Exercise 9.2: (*4 P*)

(I) Check the satisfiability of the following formulae in positive difference logic w.r.t. $\mathbb{Q}$; in case of satisfiability find a satisfying assignment.

(1) $x - y \leq 3 \ \wedge \ y - z \leq 2 \ \wedge \ x - z \leq 1 \ \wedge \ x - u \leq -3.$
(2) $x - y \leq 3 \ \wedge \ y - z \leq 2 \ \wedge \ x - z \leq 1 \ \wedge \ x - u \leq -3 \ \wedge u - x \leq 1.$
(3) $x - y \leq 3 \ \wedge \ y - z \leq 2 \ \wedge \ x - z \leq 1 \ \wedge \ x - u \leq -3 \ \wedge \ u - z \leq 3 \ \wedge \ z - x \leq 1.$

(II) Check the satisfiability of the following conjunctions in difference logic w.r.t. $\mathbb{Z}$; in case of satisfiability find a satisfying assignment.

(1) $x - y < 4 \ \wedge \ y - z \leq 2 \ \wedge \ z - x < -3 \ \wedge \ x - u \leq -3.$
(2) $x - y < 4 \ \wedge \ y - z \leq 2 \ \wedge \ z - x \leq -5 \ \wedge \ x - u < -3 \ \wedge u - x \leq 4.$
(3) $x - y < 4 \ \wedge \ y - z \leq 2 \ \wedge \ z - x < -5 \ \wedge \ x - u < -3 \ \wedge u - x \leq 4.$

(III) Check the satisfiability of the following formulae in difference logic w.r.t. $\mathbb{Q}$; in case of satisfiability find a satisfying assignment.

(1) $x - y < 4 \ \wedge \ y - z \leq 2 \ \wedge \ z - x < -5 \ \wedge \ x - u \leq -3.$
(2) $x - y < 4 \ \wedge \ y - z \leq 2 \ \wedge \ z - x \leq -6 \ \wedge \ x - u \leq -4 \ \wedge u - x \leq 4.$
(3) $x - y < 4 \ \wedge \ y - z \leq 2 \ \wedge \ z - x \leq -7 \ \wedge \ x - u < -3 \ \wedge u - x \leq 4.$

Hint: It is sufficient to check the existence of negative cycles in the associated graphs by looking at the graphs; in this assignment you do not have to use the Bellman-Ford algorithm for this.

Exercise 9.3: *(4 P)*

(I) Let F_1 be the following conjunction (in linear rational arithmetic $LI(\mathbb{Q})$):

$$F_1: \quad \begin{array}{rcll} x_1 + x_2 + 2x_3 & = & 2 & \wedge \\ x_1 + x_3 + \frac{1}{5} & < & 0 & \wedge \\ x_2 - x_3 & \leq & \frac{1}{2} & \wedge \\ x_1 + 5x_3 & \leq & 5 & \end{array}$$

Check the satisfiability of F_1 using the Fourier-Motzking method for quantifier elimination.

(II) Consider the following formulae (in linear rational arithmetic $LI(\mathbb{Q})$):

$$F_2 = \exists x \forall y \exists z (y > 0 \vee (x + y - z < 0 \wedge x + y + z < 0))$$
$$F_3 = \forall x \exists y \exists z (2x - y > 0 \wedge 2y - z > 0 \wedge z - y \geq 2 \wedge x - y < 0 \wedge y < 0)$$

Check whether F_2 and F_3 are valid or satisfiable using the Fourier-Motzkin method for quantifier elimination.

Please submit your solution until Wednesday, January 21, 2015 at 13:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework DP" in the subject.
- Put it in the box in front of Room B 222.