Decision Procedures for Verification

First-Order Logic (2)

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Exam

Question: Oral or written?

When?

1. Termin: first two weeks after end of lectures
(16.02.15-27.02.15)

2. Termin: March or April.

Doodle

Until now:

Syntax (one-sorted signatures vs. many-sorted signatures)

Signature

A signature $\Sigma = (\Omega, \Pi)$, fixes an alphabet of non-logical symbols, where

- Ω is a set of function symbols f with arity $n \geq 0$, written f/n,
- Π is a set of predicate symbols p with arity $m \geq 0$, written p/m.

A many-sorted signature $\Sigma = (S, \Omega, \Pi)$, fixes an alphabet of non-logical symbols, where

- S is a set of sorts,
- Ω is a set of function symbols f with arity $a(f) = s_1 \ldots s_n \to s$,
- Π is a set of predicate symbols p with arity $a(p) = s_1 \ldots s_m$

where $s_1, \ldots, s_n, s_m, s$ are sorts.

Variables

We assume that X is a given countably infinite set of symbols which we use for (the denotation of) variables.

Many-sorted case:

We assume that for every sort $s \in S$, X_s is a given countably infinite set of symbols which we use for (the denotation of) variables of sort s.

Terms, Atoms, Formulae

Terms over Σ (resp., Σ-terms) are formed according to these syntactic rules:

$$
\begin{array}{llll}
t, u, v & ::= & x & , x \in X & \text{(variable)} \\
 & | & f(s_1, ..., s_n) & , f/n \in \Omega & \text{(functional term)}
\end{array}
$$

Many-sorted case:

a variable $x \in X_s$ is a term of sort s

if $a(f) = s_1 \ldots s_n \to s$, and t_i are terms of sort s_i, $i = 1, \ldots, n$ then $f(t_1, ..., t_n)$ is a term of sort s.

Atoms

Atoms (also called atomic formulas) over Σ are formed according to this syntax:

$$\begin{array}{lcll} A, B & ::= & p(t_1, ..., t_m) & , \; p/m \in \Pi \\ \Big[& \mid & (t \approx t') & (\text{equation}) \quad \Big] \end{array}$$

Whenever we admit equations as atomic formulas we are in the realm of first-order logic with equality.

Many-sorted case:

If $a(p) = s_1 \ldots s_m$, we require that t_i is a term of sort s_i for $i = 1, \ldots, m$.

Equality: Several possibilities

- $\approx_s$ for every sort s
- $t \approx t'$ well-formed iff t and t' are terms of the same sort
- No restrictions (restrictions only on the semantic level)

General First-Order Formulas

$\mathsf{F}_\Sigma(X)$ is the set of first-order formulas over Σ defined as follows:

$$
\begin{array}{lcll}
F, G, H & ::= & \bot & \text{(falsum)} \\
& | & \top & \text{(verum)} \\
& | & A & \text{(atomic formula)} \\
& | & \neg F & \text{(negation)} \\
& | & (F \wedge G) & \text{(conjunction)} \\
& | & (F \vee G) & \text{(disjunction)} \\
& | & (F \rightarrow G) & \text{(implication)} \\
& | & (F \leftrightarrow G) & \text{(equivalence)} \\
& | & \forall x F & \text{(universal quantification)} \\
& | & \exists x F & \text{(existential quantification)}
\end{array}
$$

Conventions

In what follows we will use the following conventions:

constants (0-ary function symbols) are denoted with $a, b, c, d, \ldots$

function symbols with arity ≥ 1 are denoted

- $f, g, h, \ldots$ if the formulae are interpreted into arbitrary algebras
- $+, -, s, \ldots$ if the intended interpretation is into numerical domains

predicate symbols with arity 0 are denoted $P, Q, R, S, \ldots$

predicate symbols with arity ≥ 1 are denoted

- $p, q, r, \ldots$ if the formulae are interpreted into arbitrary algebras
- $\leq, \geq, <, >$ if the intended interpretation is into numerical domains

variables are denoted $x, y, z, \ldots$

Bound and Free Variables

In QxF, $Q \in \{\exists, \forall\}$, we call F the scope of the quantifier Qx.
An *occurrence* of a variable x is called bound, if it is inside the scope of a quantifier Qx.
Any other occurrence of a variable is called free.

Formulas without free variables are also called closed formulas or sentential forms.

Formulas without variables are called ground.

Bound and Free Variables

Example:

$$\forall y \quad \overbrace{(\forall \overbrace{x \quad p(x)}^{scope} \quad \rightarrow \quad q(x, y))}^{scope}$$

The occurrence of y is bound, as is the first occurrence of x. The second occurrence of x is a free occurrence.

Substitutions

Substitution is a fundamental operation on terms and formulas that occurs in all inference systems for first-order logic.

In general, substitutions are mappings

$$\sigma : X \to \mathsf{T}_\Sigma(X)$$

such that the domain of σ, that is, the set

$$dom(\sigma) = \{x \in X \mid \sigma(x) \neq x\},$$

is finite. The set of variables introduced by σ, that is, the set of variables occurring in one of the terms $\sigma(x)$, with $x \in dom(\sigma)$, is denoted by $codom(\sigma)$.

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Many-sorted case: Substitutions must be sort-preserving:
If x is a variable of sort s, then $\sigma(x)$ must be a term of sort s.

Substitutions

Substitutions are often written as $[s_1/x_1, \ldots, s_n/x_n]$, with x_i pairwise distinct, and then denote the mapping

$$[s_1/x_1, \ldots, s_n/x_n](y) = \begin{cases} s_i, & \text{if } y = x_i \\ y, & \text{otherwise} \end{cases}$$

We also write $x\sigma$ for $\sigma(x)$.

The modification of a substitution σ at x is defined as follows:

$$\sigma[x \mapsto t](y) = \begin{cases} t, & \text{if } y = x \\ \sigma(y), & \text{otherwise} \end{cases}$$

Why Substitution is Complicated

We define the application of a substitution σ to a term t or formula F by structural induction over the syntactic structure of t or F by the equations depicted on the next page.

In the presence of quantification it is surprisingly complex:
We need to make sure that the (free) variables in the codomain of σ are not *captured* upon placing them into the scope of a quantifier Qy, hence the bound variable must be renamed into a "fresh", that is, previously unused, variable z.

Application of a Substitution

"Homomorphic" extension of σ to terms and formulas:

$$f(s_1, \dots, s_n)\sigma = f(s_1\sigma, \dots, s_n\sigma)$$

$$\bot\sigma = \bot$$

$$\top\sigma = \top$$

$$p(s_1, \dots, s_n)\sigma = p(s_1\sigma, \dots, s_n\sigma)$$

$$(u \approx v)\sigma = (u\sigma \approx v\sigma)$$

$$\neg F\sigma = \neg(F\sigma)$$

$$(F\rho G)\sigma = (F\sigma\,\rho\,G\sigma) \text{ ; } \quad \text{for each binary connective } \rho$$

$$(Qx\,F)\sigma = Qz\,(F\,[x \mapsto z]\sigma) \text{ ; } \quad \text{with } z \text{ a fresh variable}$$

2.2 Semantics

To give semantics to a logical system means to define a notion of truth for the formulas. The concept of truth that we will now define for first-order logic goes back to Tarski.

As in the propositional case, we use a two-valued logic with truth values "true" and "false" denoted by 1 and 0, respectively.

Structures

A Σ-algebra (also called Σ-interpretation or Σ-structure) is a triple

$$\mathcal{A} = (U,\ (f_{\mathcal{A}} : U^n \to U)_{f/n \in \Omega},\ (p_{\mathcal{A}} \subseteq U^m)_{p/m \in \Pi})$$

where $U \neq \emptyset$ is a set, called the universe of $\mathcal{A}$.

Normally, by abuse of notation, we will have $\mathcal{A}$ denote both the algebra and its universe.

By Σ-Alg we denote the class of all Σ-algebras.

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A many-sorted Σ-algebra (also called Σ-interpretation or Σ-structure), where $\Sigma = (S, \Omega, \Pi)$ is a triple

$$\mathcal{A} = (\{U_s\}_{s \in S},\ (f_{\mathcal{A}} : U_{s_1} \times \ldots \times U_{s_n} \to U_s)_{\substack{f \in \Omega, \\ a(f) = s_1 \ldots s_n \to s}},\ (p_{\mathcal{A}} : U_{s_1} \times \ldots \times U_{s_m} \to \{0, 1\})_{\substack{p \in \Pi \\ a(p) = s_1 \ldots s_m}})$$

where $U_s \neq \emptyset$ is a set, called the universe of $\mathcal{A}$ of sort s.

Assignments

A variable has no intrinsic meaning. The meaning of a variable has to be defined externally (explicitly or implicitly in a given context) by an assignment.

A (variable) assignment, also called a valuation (over a given Σ-algebra $\mathcal{A}$), is a map $\beta : X \to \mathcal{A}$.

Variable assignments are the semantic counterparts of substitutions.

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Many-sorted case:

$\beta = \{\beta_s\}_{s \in S}, \beta_s : X_s \to U_s$

Value of a Term in $\mathcal{A}$ with Respect to β

By structural induction we define

$$\mathcal{A}(\beta) : \mathsf{T}_\Sigma(X) \to \mathcal{A}$$

as follows:

$$\begin{aligned} \mathcal{A}(\beta)(x) &= \beta(x), \qquad x \in X \\ \mathcal{A}(\beta)(f(s_1, \ldots, s_n)) &= f_{\mathcal{A}}(\mathcal{A}(\beta)(s_1), \ldots, \mathcal{A}(\beta)(s_n)), \qquad f/n \in \Omega \end{aligned}$$

Value of a Term in $\mathcal{A}$ with Respect to β

In the scope of a quantifier we need to evaluate terms with respect to modified assignments. To that end, let $\beta[x \mapsto a] : X \to \mathcal{A}$, for $x \in X$ and $a \in \mathcal{A}$, denote the assignment

$$\beta[x \mapsto a](y) := \begin{cases} a & \text{if } x = y \\ \beta(y) & \text{otherwise} \end{cases}$$

Truth Value of a Formula in $\mathcal{A}$ with Respect to β

$\mathcal{A}(\beta) : \mathsf{F}_\Sigma(X) \to \{0,1\}$ is defined inductively as follows:

$$
\begin{aligned}
\mathcal{A}(\beta)(\bot) &= 0 \\
\mathcal{A}(\beta)(\top) &= 1 \\
\mathcal{A}(\beta)(p(s_1,\ldots,s_n)) = 1 \quad &\Leftrightarrow \quad (\mathcal{A}(\beta)(s_1),\ldots,\mathcal{A}(\beta)(s_n)) \in p_{\mathcal{A}} \\
\mathcal{A}(\beta)(s \approx t) = 1 \quad &\Leftrightarrow \quad \mathcal{A}(\beta)(s) = \mathcal{A}(\beta)(t) \\
\mathcal{A}(\beta)(\neg F) = 1 \quad &\Leftrightarrow \quad \mathcal{A}(\beta)(F) = 0 \\
\mathcal{A}(\beta)(F \rho G) &= \mathsf{B}_\rho(\mathcal{A}(\beta)(F), \mathcal{A}(\beta)(G)) \\
& \text{with } \mathsf{B}_\rho \text{ the Boolean function associated with } \rho \\
\mathcal{A}(\beta)(\forall x F) &= \min_{a \in U} \{\mathcal{A}(\beta[x \mapsto a])(F)\} \\
\mathcal{A}(\beta)(\exists x F) &= \max_{a \in U} \{\mathcal{A}(\beta[x \mapsto a])(F)\}
\end{aligned}
$$

Example

The “Standard” Interpretation for Peano Arithmetic:

$$
\begin{aligned}
U_{\mathbb{N}} &= \{0, 1, 2, \ldots\} \\
0_{\mathbb{N}} &= 0 \\
s_{\mathbb{N}} &: n \mapsto n + 1 \\
+_{\mathbb{N}} &: (n, m) \mapsto n + m \\
*_{\mathbb{N}} &: (n, m) \mapsto n * m \\
\leq_{\mathbb{N}} &= \{(n, m) \mid n \text{ less than or equal to } m\} \\
<_{\mathbb{N}} &= \{(n, m) \mid n \text{ less than } m\}
\end{aligned}
$$

Note that $\mathbb{N}$ is just one out of many possible Σ_{PA}-interpretations.

Example

Values over $\mathbb{N}$ for Sample Terms and Formulas:

Under the assignment $\beta : x \mapsto 1, y \mapsto 3$ we obtain

$$
\begin{array}{lcl}
\mathbb{N}(\beta)(s(x) + s(0)) & = & 3 \\
\mathbb{N}(\beta)(x + y \approx s(y)) & = & 1 \\
\mathbb{N}(\beta)(\forall x, y(x + y \approx y + x)) & = & 1 \\
\mathbb{N}(\beta)(\forall z\ z \leq y) & = & 0 \\
\mathbb{N}(\beta)(\forall x \exists y\ x < y) & = & 1
\end{array}
$$

2.3 Models, Validity, and Satisfiability

F is valid in $\mathcal{A}$ under assignment β:

$$\mathcal{A}, \beta \models F \quad :\Leftrightarrow \quad \mathcal{A}(\beta)(F) = 1$$

F is valid in $\mathcal{A}$ ($\mathcal{A}$ is a model of F):

$$\mathcal{A} \models F \quad :\Leftrightarrow \quad \mathcal{A}, \beta \models F, \text{ for all } \beta \in X \to U_{\mathcal{A}}$$

F is valid (or is a tautology):

$$\models F \quad :\Leftrightarrow \quad \mathcal{A} \models F, \text{ for all } \mathcal{A} \in \Sigma\text{-alg}$$

F is called satisfiable iff there exist $\mathcal{A}$ and β such that $\mathcal{A}, \beta \models F$.
Otherwise F is called unsatisfiable.

Substitution Lemma

The following propositions, to be proved by structural induction, hold for all Σ-algebras $\mathcal{A}$, assignments β, and substitutions σ.

Lemma 2.3: For any Σ-term t

$$\mathcal{A}(\beta)(t\sigma) = \mathcal{A}(\beta \circ \sigma)(t),$$

where $\beta \circ \sigma : X \to \mathcal{A}$ is the assignment $\beta \circ \sigma(x) = \mathcal{A}(\beta)(x\sigma)$.

Proposition 2.4: For any Σ-formula F, $\mathcal{A}(\beta)(F\sigma) = \mathcal{A}(\beta \circ \sigma)(F)$.

Corollary 2.5: $\mathcal{A}, \beta \models F\sigma \quad \Leftrightarrow \quad \mathcal{A}, \beta \circ \sigma \models F$

These theorems basically express that the syntactic concept of substitution corresponds to the semantic concept of an assignment.

Entailment and Equivalence

F entails (implies) G (or G is a consequence of F), written $F \models G$

$:\Leftrightarrow$ for all $\mathcal{A} \in \Sigma$-alg and $\beta \in X \to U_{\mathcal{A}}$,
whenever $\mathcal{A}, \beta \models F$ then $\mathcal{A}, \beta \models G$.

F and G are called equivalent

$:\Leftrightarrow$ for all $\mathcal{A} \in \Sigma$-alg und $\beta \in X \to U_{\mathcal{A}}$ we have
$\mathcal{A}, \beta \models F \quad \Leftrightarrow \quad \mathcal{A}, \beta \models G$.

Entailment and Equivalence

Proposition 2.6:
F entails G iff $(F \rightarrow G)$ is valid

Proposition 2.7:
F and G are equivalent iff $(F \leftrightarrow G)$ is valid.

Extension to sets of formulas N in the "natural way", e.g., $N \models F$

$:\Leftrightarrow$ for all $\mathcal{A} \in \Sigma$-alg and $\beta \in X \rightarrow U_{\mathcal{A}}$:
if $\mathcal{A}, \beta \models G$, for all $G \in N$, then $\mathcal{A}, \beta \models F$.

Validity vs. Unsatisfiability

Validity and unsatisfiability are just two sides of the same medal as explained by the following proposition.

Proposition 2.8:

$$F \text{ valid} \quad \Leftrightarrow \quad \neg F \text{ unsatisfiable}$$

Hence in order to design a theorem prover (validity checker) it is sufficient to design a checker for unsatisfiability.

Q: In a similar way, entailment $N \models F$ can be reduced to unsatisfiability. How?

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Answer:

$$\begin{array}{lll} N \models F & \text{iff} & \text{there is no structure } \mathcal{A} \text{ and no assignment } \beta : X \to U_{\mathcal{A}} \\ & & \text{with } \mathcal{A}(\beta)(G) = 1 \text{ for all } G \in N \cup \{\neg F\} \\ & \text{iff} & N \cup \{\neg F\} \text{ is unsatisfiable.} \end{array}$$

Theory of a Structure

Let $\mathcal{A} \in \Sigma$-alg. The (first-order) theory of $\mathcal{A}$ is defined as

$$Th(\mathcal{A}) = \{G \in \mathsf{F}_\Sigma(X) \mid \mathcal{A} \models G\}$$

Problem of axiomatizability:

For which structures $\mathcal{A}$ can one axiomatize $Th(\mathcal{A})$, that is, can one write down a formula F (or a recursively enumerable set F of formulas) such that

$$Th(\mathcal{A}) = \{G \mid F \models G\}?$$

Analogously for sets of structures.

Two Interesting Theories

Let $\Sigma_{Pres} = (\{0/0, s/1, +/2\}, \ \emptyset)$ and $\mathbb{Z}_+ = (\mathbb{Z}, 0, s, +)$ its standard interpretation on the integers.
$Th(\mathbb{Z}_+)$ is called Presburger arithmetic (M. Presburger, 1929).
(There is no essential difference when one, instead of $\mathbb{Z}$, considers the natural numbers $\mathbb{N}$ as standard interpretation.)

Presburger arithmetic is decidable in 3EXPTIME (D. Oppen, JCSS, 16(3):323–332, 1978), and in 2EXPSPACE, using automata-theoretic methods (and there is a constant $c \geq 0$ such that $Th(\mathbb{Z}_+) \notin \text{NTIME}(2^{2^{cn}})$).

Two Interesting Theories

However, $\mathbb{N}_* = (\mathbb{N}, 0, s, +, *)$, the standard interpretation of $\Sigma_{PA} = (\{0/0, s/1, +/2, */2\}, \emptyset)$, has as theory the so-called Peano arithmetic which is undecidable, not even recursively enumerable.

Note: The choice of signature can make a big difference with regard to the computational complexity of theories.

Logical theories

Syntactic view

first-order theory: given by a set $\mathcal{F}$ of (closed) first-order Σ-formulae.

the models of $\mathcal{F}$: $\quad \mathrm{Mod}(\mathcal{F}) = \{\mathcal{A} \in \Sigma\text{-alg} \mid \mathcal{A} \models G, \text{ for all } G \text{ in } \mathcal{F}\}$

Semantic view

given a class $\mathcal{M}$ of Σ-algebras

the first-order theory of $\mathcal{M}$: $\mathrm{Th}(\mathcal{M}) = \{G \in F_\Sigma(X) \text{ closed} \mid \mathcal{M} \models G\}$

Theories

$\mathcal{F}$ set of (closed) first-order formulae

$$\mathsf{Mod}(\mathcal{F}) = \{A \in \Sigma\text{-alg} \mid \mathcal{A} \models G, \text{ for all } G \text{ in } \mathcal{F}\}$$

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$\mathsf{Th}(\mathsf{Mod}(\mathcal{F}))$ the set of formulae true in all models of $\mathcal{F}$
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Note: $\mathcal{F} \subseteq \text{Th}(\text{Mod}(\mathcal{F}))$ (typically strict)

$\mathcal{M} \subseteq \text{Mod}(\text{Th}(\mathcal{M}))$ (typically strict)

Examples

1. Groups

Let $\Sigma = (\{e/0, */2, i/1\}, \emptyset)$

Let $\mathcal{F}$ consist of all (universally quantified) group axioms:

$$\forall x, y, z \quad x * (y * z) \approx (x * y) * z$$
$$\forall x \quad x * i(x) \approx e \quad \wedge \quad i(x) * x \approx e$$
$$\forall x \quad x * e \approx x \quad \wedge \quad e * x \approx x$$

Every group $\mathcal{G} = (G, e_G, *_G, i_G)$ is a model of $\mathcal{F}$

Mod($\mathcal{F}$) is the class of all groups

$\mathcal{F} \subset \mathrm{Th}(\mathrm{Mod}(\mathcal{F}))$

Examples

2. Linear (positive)integer arithmetic

Let $\Sigma = (\{0/0, s/1, +/2\}, \{\leq /2\})$

Let $\mathbb{Z}_+ = (\mathbb{Z}, 0, s, +, \leq)$ the standard interpretation of integers.

$\{\mathbb{Z}_+\} \subset \mathsf{Mod}(\mathsf{Th}(\mathbb{Z}_+))$

3. Uninterpreted function symbols

Let $\Sigma = (\Omega, \Pi)$ be arbitrary

Let $\mathcal{M} = \Sigma$-alg be the class of all Σ-structures

The theory of uninterpreted function symbols is Th(Σ-alg) the family of all first-order formulae which are true in all Σ-algebras.

Examples

4. Lists

Let $\Sigma = (\{\text{car}/1, \text{cdr}/1, \text{cons}/2\}, \emptyset)$

Let $\mathcal{F}$ be the following set of list axioms:

$$\begin{aligned} \text{car}(\text{cons}(x, y)) &\approx x \\ \text{cdr}(\text{cons}(x, y)) &\approx y \\ \text{cons}(\text{car}(x), \text{cdr}(x)) &\approx x \end{aligned}$$

$\text{Mod}(\mathcal{F})$ class of all models of $\mathcal{F}$

$\text{Th}_{\text{Lists}} = \text{Th}(\text{Mod}(\mathcal{F}))$ theory of lists (axiomatized by $\mathcal{F}$)

Algorithmic Problems

Validity(F): $\models F$?

Satisfiability(F): F satisfiable?

Entailment(F,G): does F entail G?

Model($\mathcal{A}$,F): $\mathcal{A} \models F$?

Solve($\mathcal{A}$,F): find an assignment β such that $\mathcal{A}, \beta \models F$

Solve(F): find a substitution σ such that $\models F\sigma$

Abduce(F): find G with "certain properties" such that G entails F

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