

Exercises for “Decision Procedures for Verification” Exercise sheet 1

Exercise 1.1: (5 P)

Determine which of the following formulas are valid/satisfiable/unsatisfiable:

- (1) $(P \wedge Q) \rightarrow (P \vee Q)$
- (2) $(P \vee Q) \rightarrow (P \wedge Q)$
- (3) $\neg(P \wedge \neg\neg P)$
- (4) $Q \rightarrow \neg Q$
- (5) $Q \wedge \neg Q$
- (6) $\neg(\neg P \vee \neg\neg P)$
- (7) $((P \rightarrow Q) \wedge (\neg P \rightarrow R)) \rightarrow (Q \vee R)$

Exercise 1.2: (5 P)

If F and G are propositional formulae then prove that the following are equivalent:

- (a) $F \models G$;
- (b) $\models F \rightarrow G$ (i.e. $F \rightarrow G$ is valid);
- (c) $F \wedge \neg G$ is unsatisfiable.

Exercise 1.3: (2 P)

Prove: If N is a set of propositional formulas, then $N \models F$ if and only if $N \cup \{\neg F\}$ is unsatisfiable.

(A set of propositional formulas is unsatisfiable, if and only if for every valuation \mathcal{A} there is a formula G in the set such that $\mathcal{A} \not\models G$.)

Exercise 1.4: (5 P)

Prove:

- (1) If F_1, \dots, F_n, G are propositional formulae then $F_1 \wedge \dots \wedge F_n \rightarrow G$ is valid iff every valuation which is a model of all the formulae F_1, F_2, \dots, F_n is also a model of G .

(2) If F_1, \dots, F_n, G are propositional formulae then the following are equivalent:

- (a) $\{F_1, \dots, F_n\} \models G$
- (b) $F_1 \wedge \dots \wedge F_n \rightarrow G$ is valid
- (c) $F_1 \wedge \dots \wedge F_n \wedge \neg G$ is unsatisfiable.

Supplementary exercises

Exercise 1.5: (2 P)

Consider the formulae $F_n = \bigvee_{i=1}^n (Q_i \wedge R_i)$ for $n \in \mathbb{N}$.

As a function of n , how many clauses are in:

- (1) the CNF formula F' constructed using the distributivity of disjunctions over conjunctions?
- (2) the CNF formula F'' obtained using the satisfiability-preserving translation to clause form?
- (3) For which n is the first approach better?

You will be able to solve 1.5(2) and (3) only after the satisfiability-preserving translation to clauses normal form is presented in the lecture on Monday, 31.10.2016.

Please submit your solution until Wednesday, November 2, 2016 at 13:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to sofronie@uni-koblenz.de with the keyword “Homework DP” in the subject.
- Put it in the box in front of Room B 222.