Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Decision Procedures for Verification"
Exercise sheet 11

Exercise 11.1: *(4 P)*
Use the Nelson-Oppen procedure for checking the satisfiability of the following formulae:

1. $1 \leq c \wedge c \leq 3 \wedge f(c) \not\approx f(1) \wedge f(c) \not\approx f(3) \wedge f(1) \not\approx f(2)$
   in the combination $LI(\mathbb{Z}) \cup UIF_{\{f\}}$.
2. $f(c) \approx f(c+d) \wedge 1 \leq c \wedge c \leq d+e \wedge c+e \leq d \wedge d=1 \wedge f(c) \not\approx f(2)$
   in the combination $LI(\mathbb{Z}) \cup UIF_{\{f\}}$.

Exercise 11.2: *(4p P)*
Let $\mathcal{T} = LI(\mathbb{Q})$, and let $Q := y \leq 1, R := x \leq y, P := y+y \leq 2, S := x \geq 1$. Use a DPLL($\mathcal{T}$) method to check the satisfiability w.r.t. $\mathcal{T}$ of the following set of clauses:

$$\begin{array}{cc} (1) & \neg R \vee P \\ (2) & \neg Q \vee \neg P \\ (3) & R \vee P \\ (4) & S \end{array}$$

For checking the satisfiability of conjunctions of inequalities in $LI(\mathbb{Q})$ use the Fourier-Motzkin method.

In what follows we consider the theory of arrays defined in the lecture. We assume that the theory of indices $\mathcal{T}_i$ is $LI(\mathbb{Z})$, and the theory of elements $\mathcal{T}_e$ is $LI(\mathbb{Q})$.

Exercise 11.3: *(2 P)*
Which of the formulae below are (equivalent to formulae) in the array property fragment and which are not?
Justify your answer. (The universally quantified variables i, j are sort index; the indices k, l which are not universally quantified are considered to be constants of sort index)

(1) $\forall i \ (a[i+1] > a[i])$

(2) $\forall i \ (i < a[k] \rightarrow a[i] = a[k])$

(3) $\forall i, j \ (l_1 \leq i \leq u_1 < l_2 \leq j \leq u_2 \rightarrow a[i] \leq a[j]$

(3) $\forall i, j \ (l_1 < i \leq u_1 < l_2 \leq j \leq u_2 \rightarrow a[i] \leq a[j]$.

Supplementary exercises:

Exercise 11.4: *(5 P)*
Let $\mathcal{T}$ be a theory with signature Σ and $\mathsf{Mod}(\mathcal{T})$ be its class of models.
Show that if $\mathsf{Mod}(\mathcal{T})$ is closed under products then $\mathcal{T}$ is convex.

Exercise 11.5: *(5 P)*
We say that a theory $\mathcal{T}$ is *stably infinite* if for every quantifier-free formula ϕ, ϕ is satisfiable in $\mathcal{T}$ iff ϕ is satisfiable in a (countably) infinite model of $\mathcal{T}$.
Let $\mathcal{T}_1, \mathcal{T}_2$ be stably infinite theories with disjoint signatures. Prove that their combination $\mathcal{T}_1 \cup \mathcal{T}_2$ is stably infinite.

Please submit your solution until Wednesday, February 15, 2017 at 13:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework DP" in the subject.
- Put it in the box in front of Room B 222.