Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Decision Procedures for Verification"
Exercise sheet 2

Exercise 2.1: *(2 P)*
Determine the polarity of the following subformulae of

$$F = \neg((\neg(P \wedge \neg Q)) \vee (R \vee \neg S)) \vee (U \wedge V)$$

(1) $(P \wedge \neg Q)$

(2) Q

(3) $(R \vee \neg S)$

(4) S

(5) V

(6) $((\neg(P \wedge \neg Q)) \vee (R \vee \neg S))$

Exercise 2.2: *(4 P)*
Let F be the following formula:

$$\neg[((Q \wedge \neg P) \vee \neg(Q \vee R)) \rightarrow ((Q \rightarrow P) \wedge (Q \wedge \neg P))] \wedge (P \vee R)$$

(1) Compute the negation normal form (NNF) F' of F.

(2) Convert F' to CNF using the satisfiability-preserving transformation described in the lecture.

Exercise 2.3: *(3 P)*
Use the resolution calculus to prove that the following set of clauses is unsatisfiable:

$$\begin{array}{lc}
(1) & \neg Q \vee R \vee \neg P \vee \neg U \\
(2) & \neg Q \vee \neg P \vee S \\
(3) & P \vee \neg Q \\
(4) & \neg S \vee \neg R \\
(5) & Q \\
(6) & R \vee U
\end{array}$$

Exercise 2.4: (*3 P*)
Let $\succ$ be the ordering on propositional variables defined by $P \succ Q \succ R \succ S \succ U$. Use the ordered resolution calculus $Res^{\succ}$ to prove that the following set of clauses is unsatisfiable:

$$\begin{array}{cc}
(1) & \neg Q \vee R \vee \neg P \vee \neg U \\
(2) & \neg Q \vee \neg P \vee S \\
(3) & P \vee \neg Q \\
(4) & \neg S \vee \neg R \\
(5) & Q \\
(6) & R \vee U
\end{array}$$

You will be able to solve this exercise only after the ordered resolution calculus $Res^{\succ}$ is presented in the lecture on Monday, 7.11.2016.

Supplementary exercises (to be discussed in one of the following exercise sessions)

Exercise 2.5: (*2 P*)
Let F be a formula, P a propositional variable not occurring in F, and F' a subformula of F. Prove: The formula $F[P] \wedge (P \leftrightarrow F')$ is satisfiable if and only if $F[F']$ is satisfiable.

Here $F[F']$ is the formula F (in which F' was not replaced) and $F[P]$ is obtained from the formula F by replacing the subformula F' with the propositional variable P.

Hint: You can first prove (by induction over the formula structure of F) that for any valuation $\mathcal{A}$, if $\mathcal{A}(P) = \mathcal{A}(F')$ then $\mathcal{A}(F[P]) = \mathcal{A}(F[F'])$. This result is then used to prove the claim.

Exercise 2.6: (*4 P*)
Let F be a formula containing neither $\rightarrow$ nor $\leftrightarrow$, P a propositional variable not occurring in F, and F' a subformula of F. Prove:

- If F' has positive polarity in F then $F[F']$ is satisfiable if and only if $F[P] \wedge (P \rightarrow F')$ is satisfiable.
- If F' has negative polarity in F then $F[F']$ is satisfiable if and only if $F[P] \wedge (F' \rightarrow P)$ is satisfiable.

Please submit your solution until Wednesday, November 9, 2016 at 12:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework DP" in the subject.
- Put it in the box in front of Room B 222.

Reminder: The structural induction principle (for propositional logic).

Variant 1

Let $\mathcal{B}$ be a property of formulae in propositional logic. We want to prove that every formula over a set Π of propositional variables has property $\mathcal{B}$.

For this we proceed as follows:

Induction basis. We prove that:

- for every propositional variable $P \in \Pi$, P has property $\mathcal{B}$;
- $\bot$ and $\top$ have property $\mathcal{B}$.

Let F be a formula with propositional variables in Π.

Induction hypothesis. We assume that every strict subformula G of F has property $\mathcal{B}$.

Induction step. We use the induction hypothesis to show that also F has property $\mathcal{B}$.

We distinguish the following cases:

- $F = \neg F_1$
- $F = F_1 \ \mathsf{op} \ F_2$ for $\mathsf{op} \in \{\vee, \wedge, \rightarrow, \leftrightarrow\}$

Then property $\mathcal{B}$ holds for all Π-formulae.

Variant 2

Let $\mathcal{B}$ be a property of formulae in propositional logic. Assume that the following hold:

- for every propositional variable $P \in \Pi$, P has property $\mathcal{B}$;
- $\bot$ and $\top$ have property $\mathcal{B}$;
- if $F = F_1 \ \mathsf{op} \ F_2$ for $\mathsf{op} \in \{\vee, \wedge, \rightarrow, \leftrightarrow\}$ and if both F_1 and F_2 have property $\mathcal{B}$ then F has property $\mathcal{B}$;
- if $F = \neg F_1$ and F_1 has property $\mathcal{B}$ then F has property $\mathcal{B}$.

Then property $\mathcal{B}$ holds for all Π-formulae.