Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Decision Procedures for Verification"
Exercise sheet 3

Exercise 3.1: (*2 P*)
Assume $S \succ P \succ Q \succ R$. Let N be the following set of clauses:

$$\begin{array}{ll} (1) & \neg Q \vee \neg P \\ (2) & R \vee P \\ (3) & Q \vee S \\ (4) & \neg Q \vee \neg S \end{array}$$

(1) Which literals are maximal in the clauses of N?

(2) Which inferences are possible in the ordered resolution calculus $\mathsf{Res}^{\succ}$ with the rules:

$$\frac{C \vee A \qquad D \vee \neg A}{C \vee D} \quad \text{[ordered resolution]}$$

if C, D are clauses and A is a propositional variable with:
(i) $A \succ C$ (A is larger (in $\succ$) than the maximal literal in C);
(ii) $\neg A \succeq \max(D)$ (i.e. $\neg A$ is larger than or equal to the maximal literal of D).

$$\frac{C \vee A \vee A}{(C \vee A)} \quad \text{[ordered factoring]}$$

if C is a clause and A a propositional variable such that A is maximal in C.

- Let S be the selection function which selects the negative literal $\neg Q$ in the clauses (1) and (4). Which inferences are possible in the ordered resolution calculus with selection $\mathsf{Res}^{\succ}_{S}$ presented in the lecture.

Exercise 3.2: (*1 P*)
Find a total ordering on the propositional variables A, B, C, D, E, such that the associated clause ordering $\succ_C$ orders the clauses like this:

$$B \vee C \succ_C A \vee A \vee \neg C \succ_C C \vee E \succ_C C \vee D \succ_C \neg A \vee D \succ_C \neg E.$$

Exercise 3.3: (*4 P*)
Let N be the following set of clauses:

$$\begin{array}{ll} (1) & \neg P_3 \vee P_1 \vee P_1 \\ (2) & \neg P_2 \vee P_1 \\ (3) & P_4 \vee P_4 \\ (4) & P_4 \\ (5) & P_3 \vee \neg P_2 \\ (6) & P_4 \vee P_3 \end{array}$$

(1) Let $\succ$ be the ordering on propositional variables defined by $P_4 \succ P_3 \succ P_2 \succ P_1$. Sort the clauses in N according to $\succ_C$. Which literals are maximal in the clauses of N?
(2) Define a selection function S such that N is saturated under $Res^{\succ}_S$.
(3) Construct a model of N using the canonical construction presented in the lecture.

Exercise 3.4: (*2 P*)
Use a DPLL procedure to find a model of each of the following formulae, or prove that the particular formula has no model:

(1) $(P \vee \neg Q) \wedge (\neg P \vee Q) \wedge (Q \vee \neg R) \wedge (\neg Q \vee \neg R)$
(2) $(P \vee Q \vee \neg R) \wedge (P \vee \neg Q) \wedge (P \vee Q \vee R) \wedge (R \vee Q) \wedge (R \vee \neg Q) \wedge (\neg P \vee \neg R) \wedge \neg U$

You will be able to solve this exercise only after the DPLL procedure is presented in the lecture on Monday, 14.11.2016.

Please submit your solution until Wednesday, November 16, 2016 at 11:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework DP" in the subject.
- Put it in the box in front of Room B 222.