Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Decision Procedures for Verification"
Exercise sheet 5

Exercise 5.1: *(3 P)*
Let $\Sigma = \{0, s, +\}$. Consider the following formulae in the signature Σ:

1. $F_1 = \forall x\ (x + 0 \approx x)$
2. $F_2 = \forall x, y\ (x + s(y) \approx s(x + y))$
3. $F_3 = \forall x, y\ \ (x + y \approx y + x)$.

Find a Σ-structure in which F_1 and F_2 are valid but F_3 is not.

Exercise 5.2: *(2 P)*
Compute a clausal normal form for the following formula:

$$\exists x\, \forall y\, (\forall z\, (p(y, z) \vee \neg\, x \approx y) \rightarrow (\forall z\, q(y, z) \wedge \neg\, r(x, y)))$$

Exercise 5.3: *(3 P)*
Consider the following formulae:

- $F_1 := \forall x(S(x) \rightarrow \exists y(R(x, y) \wedge P(y)))$
- $F_2 := \forall x(P(x) \rightarrow Q(x))$
- $F_3 := \exists x S(x)$
- $G := \exists x \exists y(R(x, y) \wedge Q(y))$

Use the resolution calculus to prove that $\{F_1, F_2, F_3\} \models G$.

Supplementary exercise

Exercise 5.4: *(4 P)*
Let $\Sigma = (\Omega, \Pi)$ be a signature and X a set of variables. Let $\mathcal{A}$ be a Σ-structure and $\beta : X \rightarrow U_{\mathcal{A}}$ a variable assignment.

(1) Prove that for every formula $F \in F_\Sigma(X)$ and every $x \in X$, the truth values $\mathcal{A}(\beta)(\forall x F)$ and $\mathcal{A}(\beta)(\exists x F)$ do not depend on $\beta(x)$.

(2) Use (1) to show that if G is a closed formula in $F_\Sigma(X)$, then the truth value of G in $\mathcal{A}$ w.r.t. β, $\mathcal{A}(\beta)(G)$ does not depend on the way β is defined.

(3) Use (2) to prove that $\mathsf{Th}(\mathsf{Mod}(\mathcal{F})) = \{G \in F_\Sigma(X) \text{ closed} \mid \mathcal{F} \models G\}$.

Please submit your solution until Wednesday, November 31, 2016 at 12:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework DP" in the subject.
- Put it in the box in front of Room B 222.