Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Decision Procedures for Verification"
Exercise sheet 6

Exercise 6.1: (*4 P*)
Let $\Sigma = (\Omega, \Pi)$ with $\Omega = \{b/0, f/1\}$ and $\Pi = \{p/1\}$.

(a) Which is the universe of the Herbrand interpretations over this signature?
If $\mathcal{A}$ is a Herbrand interpretation over Σ how are $b_{\mathcal{A}}$ and $f_{\mathcal{A}}$ defined?

(b) How many different Herbrand interpretations over Σ do exist? Explain briefly.

(c) How many different Herbrand models over Σ does the formula:

$$p(f(f(b))) \wedge \forall x(p(x) \rightarrow p(f(x))) \tag{1}$$

have? Explain briefly.

(d) Every Herbrand model over Σ of (1) is also a model of

$$\forall x p(f(f(x))) \tag{2}$$

Give an example of an algebra that is a model of (1) but not of (2).

Exercise 6.2: (*1 P*)
Which of the following formulae is in the Bernays-Schönfinkel class?

(1) $\exists y \forall x \exists z \quad ((p(x) \vee q(y)) \wedge (p(z) \vee \neg q(y))$

(2) $\forall x \exists y \forall z \exists u((p(x) \vee q(y)) \wedge (q(y) \vee r(u, x))$

(3) $\exists y \exists z \forall x[(p(x) \vee q(y)) \wedge q(z)]$

Exercise 6.3: (*2 P*)
Compute a most general unifier of

$$\{\, f(x, g(x)) = y,\ h(y) = h(v),\ v = f(g(z), w) \,\}$$

using the method presented in the lecture.

Exercise 6.4: (*3 P*)
Let $\succ$ be a total and well-founded ordering on ground atoms such that, if the atom A contains more symbols than B, then $A \succ B$. Let N be the following set of clauses:

$$\neg q(z,z)$$
$$\neg q(f(x),y) \vee q(f(f(x)),y) \vee p(x)$$
$$\neg p(a) \vee \neg p(f(a)) \vee q(f(a), f(f(a)))$$
$$p(f(x)) \vee p(g(y))$$
$$\neg p(g(a)) \vee p(f(f(a)))$$

(a) Which literals are maximal in the clauses of N?
(b) Define a selection function S such that N is saturated under $Res_S^{\succ}$. Justify your choice.

Remark: You will be able to solve this exercise only after the lecture from 5.12.2016.

Supplementary exercise

Exercise 6.5: *(3 P)*
Let F be a closed first-order formula with equality over a signature $\Sigma = (\Omega, \Pi)$. Let $\sim \notin \Omega$ be a new binary relation symbol (written as an infix operator). Let the set $Eq(\Sigma)$ contain the formulas

$$\forall x\,(x \sim x)$$
$$\forall x, y\,(x \sim y \rightarrow y \sim x)$$
$$\forall x, y, z\,(x \sim y \wedge y \sim z \rightarrow x \sim z)$$

and for every $f/n \in \Omega$ the formula

$$\forall x_1, \ldots, x_n, y_1, \ldots, y_n\,(x_1 \sim y_1 \wedge \cdots \wedge x_n \sim y_n \rightarrow f(x_1, \ldots, x_n) \sim f(y_1, \ldots, y_n))$$

and for every $p/n \in \Pi$ the formula

$$\forall x_1, \ldots, x_n, y_1, \ldots, y_n\,(x_1 \sim y_1 \wedge \cdots \wedge x_n \sim y_n \wedge p(x_1, \ldots, x_n) \rightarrow p(y_1, \ldots, y_n)).$$

Let $\tilde{F}$ be the formula that one obtains from F if every occurrence of the equality symbol $\approx$ is replaced by the relation symbol $\sim$.

(a) *Definition.* A binary relation $\sim$ on the support of a Σ-algebra satisfying all properties in $Eq(\Sigma)$ is called a congruence relation.

Let $\mathcal{A}$ be a model of $\tilde{F} \cup Eq(\Sigma)$. Show that the interpretation $\sim_{\mathcal{A}}$ of $\sim$ in $\mathcal{A}$ is a congruence relation. (It is enough if you prove *one* of the properties of congruence relations, say symmetry; the other properties are proved analogously.)

(b) Let $\mathcal{A}$ be a model of $\tilde{F} \cup Eq(\Sigma)$. Use the congruence relation $\sim_{\mathcal{A}}$ to construct a model of F and prove that it is a model.

(c) Prove that a formula F is satisfiable if and only if $Eq(\Sigma) \cup \{\tilde{F}\}$ is satisfiable.

Please submit your solution until Wednesday, December 7, 2016 at 12:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework DP" in the subject.
- Put it in the box in front of Room B 222.