Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Decision Procedures for Verification"
Exercise sheet 7

Exercise 7.1: *(2 P)*
Redundant clauses remain redundant, if the theorem prover deletes redundant clauses. Prove: If N and M are sets of clauses and $M \subseteq \text{Red}(N)$, then $\text{Red}(N) \subseteq \text{Red}(N \backslash M)$.

Exercise 7.2: *(5 P)*
Assume $S \succ P \succ Q \succ R$. Let N be the following set of clauses:

$$
\begin{array}{ll}
(1) & \neg Q \vee P \vee R \\
(2) & \neg R \vee P \\
(3) & Q \vee S \vee \neg P \\
(4) & \neg Q \vee \neg S
\end{array}
$$

Give the definition of redundancy of a clause w.r.t. a set of clauses.
Is the clause $\neg Q \vee P \vee S$ redundant w.r.t. the set N above?
Is the clause $\neg Q \vee P$ redundant w.r.t. the set N above? Justify your answer.

Assume $U \succ S \succ P \succ Q \succ R$. Let N be the following set of clauses:

$$
\begin{array}{ll}
(1) & \neg Q \vee P \vee R \\
(2) & \neg R \vee P \\
(3) & \neg Q \vee P \vee S \\
(4) & Q \vee S \vee \neg P \\
(5) & \neg Q \vee \neg S
\end{array}
$$

Is the clause $\neg Q \vee P \vee S \vee U$ redundant w.r.t. the set consisting of the clauses (1), (2), (4) and (5)? Justify your answer.

Exercise 7.3: *(2 P)*
To which of the classes discussed in the lecture (the Bernays-Schönfinkel class, the Ackermann class or the monadic class) do the following formulae belong:

(1) $\exists y \forall x \ \ ((p(x) \vee r(x, y)) \wedge q(y))$
(2) $\forall x \exists y \forall z \exists u ((p(x) \vee q(y)) \wedge (q(y) \vee p(u))$
(3) $\exists z \forall x \exists y (p(x) \vee q(y)) \wedge q(z)$
(4) $\exists x \forall y (P(x) \vee R(y)) \wedge Q(y)$

Note that they can be in more than one, or in none of the classes above.

Exercise 7.4: *(2 P)*
Let F and G be propositional formulae over $\Pi = \{P, Q, R, S, T, U\}$ such that:

- The CNF of F is the following set N of clauses:

$$\begin{array}{rc} (1) & P \vee Q \\ (2) & \neg P \vee R \vee S \\ (3) & \neg P \vee \neg R \\ (4) & P \vee U \end{array}$$

- The CNF of $\neg G$ consists of the set M of clauses:

$$\begin{array}{rc} (5) & R \vee \neg S \\ (6) & \neg R \vee Q \\ (7) & \neg Q \vee R \\ (8) & \neg S \vee T \\ (9) & S \vee \neg T \\ (10) & \neg Q \vee \neg R \end{array}$$

Which propositional variables occur only in N and not in M?

Which propositional variables occur both in N and in M?

Use the method described in the lecture to construct a Craig interpolant for $F \models G$.

Exercise 7.5: *(7 P)*
Let $\Sigma = (\{c_1/0, \dots, c_n/0, f_1/1, \dots, f_n/1\}, \Pi)$ be a signature. Consider the following classes of clauses:

- G (denoted in the lecture also $G(c_1, \dots, c_n)$) is the class of all ground clauses in the signature Σ which do not contain any occurrence of a unary function symbol.
- V (denoted in the lecture also $V(x, c_1, \dots, c_n)$) is the class of all clauses with one variable (x) in the signature Σ which do not contain any occurrence of a unary function symbol.
- G_f (denoted in the lecture also $G(c_1, \dots, c_n, f_k(c_j))$) is the class of all ground clauses in the signature Σ which contain at least one occurrence of a unary function symbol (having as argument a constant); no nested applications of unary function symbols are allowed.

  Example: Assume $p/3.q/2 \in \Pi$ Then:
  $C_1 : p(c_1, c_2, c_3) \vee \neg q(c_2, c_1) \notin G_f$
  $C_2 : q(c_1, c_2) \vee \neg q(f_1(c_3), c_4) \in G_f$
  $C_3 : q(c_1, c_2) \vee \neg q(f_1(c_3), f_2(f_3(c_4))) \notin G_f$.

- V_f (denoted in the lecture also $V(x, c_1, \dots, c_n, f_j(x))$) is the class of all ground clauses in the signature Σ which contain only one variable (x), at least one occurrence of a unary function symbol (having as argument the variable x), no occurrences of terms of the form $f_k(c_j)$; in addition no nested applications of unary function symbols are allowed.

Example: Assume $p/3.q/2 \in \Pi$ Then:
$C_1' : p(x, c_2, x) \vee \neg q(c_2, c_1) \notin G_f$
$C_2' : q(x, c_2) \vee \neg q(f_1(c_3), x) \notin G_f$
$C_3' : q(c_1, x) \vee \neg q(x, f_2(f_3(x))) \notin G_f$ $C_4' : p(x, c_2, x) \vee \neg p(c_2, x, f(x)) \in G_f$.

Consider a term ordering $\succ$ in which $f(t) \succ t$ for every term t and terms containing function symbols of arity 1 are larger than those who do not. Consider the general ordered resolution calculus $\mathsf{Res}^{\succ}$. Prove that in this calculus:

(1) The resolvent of a clause in G_f and a clause in V_f is a clause in G_f of G.

(2) The resolvent of two clauses in V_f is a clause in G, G_f, V or V_f.

Show that (up to renaming the variables) the number of different clauses in the set $G \cup V \cup G_f \cup V_f$ is finite.

Please submit your solution until Wednesday, December 14, 2016 at 13:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework DP" in the subject.
- Put it in the box in front of Room B 222.