

### Exercises for “Decision Procedures for Verification” Exercise sheet 8

#### Exercise 8.1: (2 P)

Let  $\phi$  be the following (ground) formula:

$$f(f(c)) \approx f(c) \wedge f(f(c)) \approx f(d) \wedge d \not\approx f(c).$$

- (1) Compute  $FLAT(\phi)$  (the formula obtained by recursively replacing, in a bottom-up fashion, any term of the form  $f(c')$ , where  $c'$  is a constant, with a new constant).
- (2) Compute  $FC(\phi)$  (the set of functional consistency axioms associated with the flattening above):

$$FC(\phi) = \{c_1 \approx c_2 \rightarrow d_1 \approx d_2 \mid d_i \text{ is introduced as an abbreviation for } f(c_i)\}.$$

- (3) Check whether  $FLAT(\phi) \wedge FC(\phi)$  is satisfiable.
- (4) Is  $\phi$  is satisfiable? Justify your answer.

#### Exercise 8.2: (6 P)

Check the satisfiability of the following ground formulae using the algorithm based on congruence closure presented in the lecture.

- (1)  $\phi_1 = f(f(c)) \approx f(c) \wedge f(f(c)) \approx f(d) \wedge d \not\approx f(c).$
- (2)  $\phi_2 = f(f(c)) \approx f(c) \wedge f(c) \approx d \wedge f(d) \not\approx f(f(c)).$

#### Exercise 8.3: (6 P)

Check the satisfiability of the following formulae in positive difference logic w.r.t.  $\mathbb{Q}$ ; in case of satisfiability find a satisfying assignment.

- (1)  $x - y \leq 3 \wedge y - z \leq 2 \wedge x - z \leq 1 \wedge x - u \leq -3.$
- (2)  $x - y \leq 3 \wedge y - z \leq 2 \wedge x - z \leq 1 \wedge x - u \leq -3 \wedge u - x \leq 1.$
- (3)  $x - y \leq 3 \wedge y - z \leq 2 \wedge x - z \leq 1 \wedge x - u \leq -3 \wedge u - z \leq 3 \wedge z - x \leq 1.$

Please submit your solution until Wednesday, January 15, 2014 at 13:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to [sofronie@uni-koblenz.de](mailto:sofronie@uni-koblenz.de) with the keyword “Homework DP” in the subject.
- Put it in the box in front of Room B 222.