Let $\mathcal{T}$ be the combination of $LI(\mathbb{Q})$ (linear arithmetic over $\mathbb{Q}$) and UIF_Σ, the theory of uninterpreted function symbols in a signature Σ containing the unary function h.

Check the satisfiability of the following ground formula w.r.t. $\mathcal{T}$ using the deterministic version of the Nelson-Oppen procedure (after purifying the formulae check for entailment of equalities between shared constants and propagate the entailed equalities):

- $\phi: \quad a+h(h(h(b))+c) \approx e \ \wedge \ h(b) \approx b' \ \wedge \ e \approx h(b')+c \ \wedge \ a+h(e) \not\approx e.$

Solution
Step 1: Purification

$$a + \underbrace{h(\underbrace{\underbrace{h(h(b))}_{b_1}+c}_{c_1})}_{b_2} \approx e \quad \wedge \quad h(b) \approx b' \quad \wedge \quad e \approx \underbrace{h(b')}_{b''}+c \quad \wedge \quad a + \underbrace{h(e)}_{e_1} \not\approx e.$$

$LI(\mathbb{Q})$	UIF
$a + b_2 \approx e$	$h(h(b)) \approx b_1$
$b_1 + c \approx c_1$	$h(c_1) \approx b_2$
$e \approx b'' + c$	$h(b) \approx b'$
$a + e_1 \not\approx e$	$h(b') \approx b''$
	$h(e) \approx e_1$
ϕ_1	ϕ_2

Both ϕ_1 and ϕ_2 are satisfiable, so we start Step 2.

Step 2: Propagation

Shared constants: $\{b_1, b_2, b'', e, e_1\}$.

We notice that $\phi_2 \models_{UIF} b_1 \approx b''$

Inded, as $b(b) \approx b'$ we have $h(h(b)) \approx h(b')$. ϕ_1 contains $h(h(b)) \approx b_1 \wedge h(b') \approx b''$, thus $\phi_2 \models_{UIF} b_1 \approx b''$.

We propagate $b_1 \approx b''$ and obtain:

$LI(\mathbb{Q})$	UIF
$a + b_2 \approx e$	$h(h(b)) \approx b_1$
$b_1 + c \approx c_1$	$h(c_1) \approx b_2$
$e \approx b'' + c$	$h(b) \approx b'$
$a + e_1 \not\approx e$	$h(b') \approx b''$
	$h(e) \approx e_1$
$b_1 \approx b''$	$b_1 \approx b''$
ϕ_1^1	ϕ_2^1

We notice that $\phi_1^1 \models_{LI(Q)} e \approx c_1$

> Inded, as $b_1 \approx b''$ we have $b_1 + c \approx b'' + c$, so since ϕ_1^1 contains $b_1 + c \approx c_1$ and $b'' + c \approx e$ we obtain $\phi_1^1 \models e \approx c_1$.

We propagate $e \approx c_1$ and obtain:

$LI(\mathbb{Q})$	UIF
$a + b_2 \approx e$	$h(h(b)) \approx b_1$
$b_1 + c \approx c_1$	$h(c_1) \approx b_2$
$e \approx b'' + c$	$h(b) \approx b'$
$a + e_1 \not\approx e$	$h(b') \approx b''$
	$h(e) \approx e_1$
$b_1 \approx b''$	$b_1 \approx b''$
$e \approx c_1$	$e \approx c_1$
ϕ_1^2	ϕ_2^2

We notice that $\phi_2^2 \models_{UIF} b_2 \approx e_1$

> Inded, as $e \approx c_1$ we have $h(e) \approx h(c_1)$. ϕ_2^2 contains $h(e) \approx e_1 \wedge h(c_1) \approx b_2$, thus $\phi_2 \models_{UIF} b_2 \approx e_1$.

We propagate $b_2 \approx e_1$ and obtain:

$LI(\mathbb{Q})$	UIF
$a + b_2 \approx e$	$h(h(b)) \approx b_1$
$b_1 + c \approx c_1$	$h(c_1) \approx b_2$
$e \approx b'' + c$	$h(b) \approx b'$
$a + e_1 \not\approx e$	$h(b') \approx b''$
	$h(e) \approx e_1$
$b_1 \approx b''$	$b_1 \approx b''$
$e \approx c_1$	$e \approx c_1$
$b_2 \approx e_1$	$b_2 \approx e_1$
ϕ_1^3	ϕ_2^3

We now show that ϕ_1^3 is unsatisfiable:
,

From $b_2 \approx e_1$ it follows that $a + b_2 \approx a + e_1$.
But ϕ_1^3 contains $a + b_2 \approx e$ and $a + e_1 \not\approx e$. Contradiction.

Thus, the formula ϕ is unsatisfiable.