Exercise 4.6 Let N be a set of clauses in propositional logic with the property that every clause consists of two literals. Prove that the satisfiability of N can be checked in polynomial time in the size of N.

Solution: Assume N contains propositional variables in a set $\{P_1 \dots P_n\}$. Then every clause in $Res^*(N)$ will only contain propositional variables in $\{P_1 \dots P_n\}$.

We show that there is a polynomial bound (in n) on the number of clauses in $Res^*(N)$ and hence, that resolution and factoring need to be applied only polynomially many times (in n) to derive the empty clause or to prove that N is satisfiable.

From propositional variables $\{P_1 \dots P_n\}$ we can build $2n$ literals $(P_1, \neg P_1, P_2, \neg P_2, \dots P_n, \neg P_n)$.
From these $2n$ literals we can build $(2n)\cdot(2n) = 4n^2$ clauses with two literals and $2n$ clauses with one literal, that is $4n^2+2n$ clauses with 1 or 2 literals.

The resolvent of two clauses with 1 or 2 literals can be the empty clause or a clause with 1 or 2 literals. Factorization yields a clause with 1 literal from a clause with two (equal) positive literals.

Thus, we can apply at most $(4n^2+2n)^2$ resolution steps and at most $2n$ factorization steps on clauses with 1 or 2 literals over variables $\{P_1 \dots P_n\}$.
Thus, the set N of clauses can be saturated under the resolution calculus in polynomial time in the number of variables (and, hence, in the size of N).