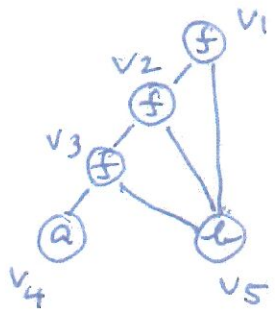
Task Check the satisfiability of the following set of ground formulae: in UIF

$$F: f(a,b) \simeq a \wedge f(f(a,b),b) \simeq a \wedge f(f(f(a,b),b),b) \not\simeq a$$

Solution: 1) CONSTRUCT DAG



$v_1 : f(f(f(a,b),b),b)$

$v_2 : f(f(a,b),b)$

$v_3 : f(a,b)$

$v_4 : a$

$v_5 : b$

$R = \{(v_3, v_4), (v_2, v_4)\}$

Task

Decide whether $(v_1, v_4) \in R^c$

if $(v_1, v_4) \in R^c$: F unsatisfiable

if $(v_1, v_4) \notin R^c$: F satisfiable

2) COMPUTE R^c

$R_0 := Id$

$R_1 := MERGE(v_3, v_4)$

- Test representatives
  $FIND(v_3) = v_3 \neq v_4 = FIND(v_4)$
- $P_{v_3} = \{v_2\}$, $P_{v_4} = \{v_3\}$
- Merge congruence classes:

(*) UNION(v_3, v_4): sets $FIND(v_3)$ to $FIND(v_4) = v_4$.

- Compute and recursively merge predecessors $P_{v_3} = \{v_2\}$, $P_{v_4} = \{v_3\}$
  - merge if necessary v_2 and v_3
  - $FIND(v_2) = v_2 \neq v_4 = FIND(v_3)$. Test whether CONGRUENT$(v_2, v_3)$
    MERGE(v_2, v_3):
    - $P_{v_2} = \{v_1\}$, $P_{v_3} = \{v_3, v_2\}$.
    - Merge congruence classes:

      (**) UNION(v_2, v_3): sets $FIND(v_2)$ to $FIND(v_3) = v_4$.

    - Compute and merge predecessors $P_{v_2} = \{v_1\}$; $P_{v_3} = \{v_3, v_2\}$
      - merge, if necessary v_1 and v_2
      - $FIND(v_1) = v_1 \neq v_4 = FIND(v_2)$. Test whether CONGRUENT$(v_1, v_2)$ (same label, successors same FIND)
        MERGE(v_1, v_4).
      - $P_{v_1} = \emptyset$, $P_{v_2} = \{v_3, v_2, v_1\}$
      - Merge congruence classes:

        (***) UNION(v_1, v_2): sets $FIND(v_1)$ to $FIND(v_2) = v_4$.

      - Compute and merge predecessors: $P_{v_1} = \emptyset$: nothing to do

CONGRUENT(v_2, v_3): yes: $\lambda(v_2) = \lambda(v_3) = f$
$FIND(v_2[1]) = FIND(v_3[1])$ (v_3, v_4)
$FIND(v_2[2]) = FIND(v_3[2])$ (v_5, v_5)

	id. FIND	(*) FIND	(**) FIND	(***) FIND
v_1	v_1	v_1	v_1	v_4
v_2	v_2	v_2	v_4	v_4
v_3	v_3	v_4	v_4	v_4
v_4	v_4	v_4	v_4	v_4
v_5	v_5	v_5	v_5	v_5

R_1: corresponds to partition described by (***) FIND

$R_2 := MERGE(v_2, v_4)$

- Test representatives: $FIND(v_2) = v_4 = FIND(v_4)$: Do not need to merge (already merged)

$\Rightarrow R^c = R_1$. We can see that $(v_1, v_4) \in R^c$, so F is unsatisfiable.