

Exercises for “Decision Procedures for Verification” Exercise sheet 3

Exercise 3.1: (2 P)

Assume $P \succ Q \succ R$. Let N be the following set of clauses:

- (1) $\neg R \vee P$
- (2) $\neg Q \vee \neg P$
- (3) Q
- (4) $R \vee P$

Let S be the selection function which selects $\neg R$ in clause (1) and $\neg Q$ in clause (2).

Use the ordered resolution calculus with selection $\text{Res}_S^>$ described in the lecture for checking the satisfiability of the set N of clauses.

Exercise 3.2: (2 P)

A propositional Horn clause is a clause which has at most one positive literal.

(*Example:* $\neg P \vee Q \vee \neg R$, $\neg P \vee \neg R$ and Q are Horn clauses,
whereas $\neg P \vee Q \vee R$ and $Q \vee R$ are not Horn clauses.)

Prove: Every set H of clauses with the following properties:

- (i) H consists only of Horn clauses;
- (ii) Every clause in H contains at least one negative literal;

is satisfiable.

Exercise 3.3: (5 P)

Let H be a set of propositional Horn clauses. The size of H is the number of all literals which occur in H . Prove that the resolution calculus $\text{Res}_S^>$ (or the marking algorithm discussed in the lecture “Logik für Informatiker”) can check the satisfiability of H in time polynomial in the size of H .

Supplementary question: Can you give an algorithm for check the satisfiability of H in time linear in the size of H ?

Exercise 3.4: (2 P)

Use a DPLL procedure to find a model of each of the following formulae, or prove that the particular formula has no model:

(1) $(P \vee \neg Q) \wedge (\neg P \vee Q) \wedge (Q \vee \neg R) \wedge (\neg Q \vee \neg R)$

(2) $(P \vee Q \vee \neg R) \wedge (P \vee \neg Q) \wedge (P \vee Q \vee R) \wedge (R \vee Q) \wedge (R \vee \neg Q) \wedge (\neg P \vee \neg R) \wedge \neg U$

Please submit your solution until Wednesday, November 14, 2018 at 16:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to sofronie@uni-koblenz.de with the keyword “Homework DP” in the subject.
- Put it in the box in front of Room B 222.