Decision Procedures for Verification

Combinations of Decision Procedures (3)

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Last time

Combinations of Decision Procedures

The Nelson/Oppen Procedure

(for theories with disjoint signature)

From conjunctions to arbitrary combinations

DPLL(T)

Satisfiability of formulae with quantifiers

Satisfiability of formulae with quantifiers

In many applications we are interested in testing the satisfiability of formulae containing (universally quantified) variables.

Examples

- check satisfiability of formulae in the Bernays-Schönfinkel class
- check whether a set of (universally quantified) Horn clauses entails a ground clause
- check whether a property is consequence of a set of axioms

  Example 1: $f : \mathbb{Z} \to \mathbb{Z}$ is monotonely increasing
  and $g : \mathbb{Z} \to \mathbb{Z}$ is defined by $g(x) = f(x + x)$
  then g is also monotonely increasing

  Example 2: If array a is increasingly sorted, and
  x is inserted before the first position i with $a[i] > x$
  then the array remains increasingly sorted.

A theory of arrays

We consider the theory of arrays in a many-sorted setting.

Syntax:

- Sorts: Elem (elements), Array (arrays) and Index (indices, here integers).
- Function symbols: read, write.

$$a(\text{read}) = \mathit{Array} \times \mathit{Index} \to \mathit{Element}$$

$$a(\text{write}) = \mathit{Array} \times \mathit{Index} \times \mathit{Element} \to \mathit{Array}$$

Theories of arrays

We consider the theory of arrays in a many-sorted setting.

Theory of arrays $\mathcal{T}_{arrays}$:

- $\mathcal{T}_i$ (theory of indices): Presburger arithmetic
- $\mathcal{T}_e$ (theory of elements): arbitrary
- Axioms for read, write

$$\begin{array}{rcl} read(write(a, i, e), i) & \approx & e \\ j \not\approx i \vee read(write(a, i, e), j) & = & read(a, j). \end{array}$$

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\begin{array}{rcl}
read(write(a, i, e), i) & \approx & e \\
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\end{array}
$$

Fact: Undecidable in general.

Goal: Identify a fragment of the theory of arrays which is decidable.

A decidable fragment

- **Index guard** a positive Boolean combination of atoms of the form $t \leq u$ or $t = u$ where t and u are either a variable or a ground term of sort Index

  Example: $(x \leq 3 \vee x \approx y) \wedge y \leq z$ is an index guard

  Example: $x + 1 \leq c, \quad x + 3 \leq y, \quad x + x \leq 2$ are not index guards.

- **Array property formula** [Bradley,Manna,Sipma'06]

  $(\forall i)(\varphi_I(i) \rightarrow \varphi_V(i))$, where:

  φ_I: index guard

  φ_V: formula in which any universally quantified i occurs in a direct array read; no nestings

  Example: $c \leq x \leq y \leq d \rightarrow a(x) \leq a(y)$ is an array property formula

  Example: $x < y \rightarrow a(x) < a(y)$ is not an array property formula

Decision Procedure

(Rules should be read from top to bottom)

Step 1: Put F in NNF.

Step 2: Apply the following rule exhaustively to remove writes:

$$\frac{F[\mathit{write}(a, i, v)]}{F[a'] \wedge a'[i] = v \wedge (\forall j.\, j \neq i \rightarrow a[j] = a'[j])} \quad \text{for fresh } a' \text{ (write)}$$

Given a formula F containing an occurrence of a write term $\mathit{write}(a, i, v)$, we can substitute every occurrence of $\mathit{write}(a, i, v)$ with a fresh variable a' and explain the relationship between a' and a.

Decision Procedure

Step 3 Apply the following rule exhaustively to remove existential quantification:

$$\frac{F[\exists i.\, G[i]]}{F[G[j]]} \text{ for fresh } j \text{ (exists)}$$

Existential quantification can arise during Step 1 if the given formula contains a negated array property.

Decision Procedure

Steps 4-6 accomplish the reduction of universal quantification to finite conjunction.

The main idea is to select a set of symbolic index terms on which to instantiate all universal quantifiers.

Theories of arrays

Step 4 From the output F3 of Step 3, construct the index set $\mathcal{I}$:

$$\begin{aligned}\mathcal{I} \quad = \quad & \{\lambda\}\cup \\ & \{t \mid \cdot[t] \in F3 \text{ such that } t \text{ is not a universally quantified variable}\}\cup \\ & \{t \mid t \text{ occurs as an } \textit{evar} \text{ in the parsing of index guards}\}\end{aligned}$$

(evar is any constant, ground term, or unquantified variable.)

This index set is the finite set of indices that need to be examined. It includes all terms t that occur in some $\textit{read}(a, t)$ anywhere in F (unless it is a universally quantified variable) and all terms t that are compared to a universally quantified variable in some index guard.

λ is a fresh constant that represents all other index positions that are not explicitly in $\mathcal{I}$.

Theories of arrays

Step 5 Apply the following rule exhaustively to remove universal quantification:

$$\frac{H[\forall \bar{i}. F[i] \rightarrow G[i]]}{H\left[\bigwedge_{\bar{i} \in \mathcal{I}^n} (F[\bar{i}] \rightarrow G[\bar{i}])\right]} \quad \text{(forall)}$$

where n is the size of the list of quantified variables $\bar{i}$.

This is the key step.

It replaces universal quantification with finite conjunction over the index set. The notation $\bar{i} \in \mathcal{I}^n$ means that the variables $\bar{i}$ range over all n-tuples of terms in $\mathcal{I}$.

Theories of arrays

Step 6: From the output $F5$ of Step 5, construct

$$F6: \qquad F5 \wedge \bigwedge_{i \in \mathcal{I} \setminus \{\lambda\}} \lambda \neq i$$

The new conjuncts assert that the variable λ introduced in Step 4 is unique: it does not equal any other index mentioned in F5.

Step 7: Decide the TA-satisfiability of $F6$ using the decision procedure for the quantifier free fragment.

Example

Consider the array property formula

$$F : write(a, l, v)[k] = b[k] \wedge b[k] \neq v \wedge a[k] = v \wedge (\forall i. i \neq l \rightarrow a[i] = b[i])$$

It contains one array property,

$$\forall i. i \neq l \rightarrow a[i] = b[i]$$

index guard: $i \neq l := (i \leq l - 1 \vee i \geq l + 1)$ value constraint: $a[i] = b[i]$

Step 1: The formula is already in NNF.

Step 2: We rewrite F as:

$$\begin{aligned} F2 : \quad & a'[k] = b[k] \wedge b[k] \neq v \wedge a[k] = v \wedge (\forall i. i \neq l \rightarrow a[i] = b[i]) \\ & \wedge a'[l] = v \wedge (\forall j. j \neq l \rightarrow a[j] = a'[j]). \end{aligned}$$

Example

Consider the array property formula

$$F : write(a, l, v)[k] = b[k] \wedge b[k] \neq v \wedge a[k] = v \wedge (\forall i. i \neq l \rightarrow a[i] = b[i])$$

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index guards: $i \neq l := (i \leq l - 1 \vee i \geq l + 1)$ value constraint: $a[i] = b[i]$

$j \neq l := (j \leq l - 1 \vee j \geq l + 1)$ value constraint: $a[i] = a'[j]$

Step 3: F2 does not contain any existential quantifiers $\mapsto$ F3 = F2.

Step 4: The index set is

$$\mathcal{I} = \{\lambda\} \cup \{k\} \cup \{l, l - 1, l + 1\} = \{\lambda, k, l, l - 1, l + 1\}$$

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$$F : write(a, l, v)[k] = b[k] \wedge b[k] \neq v \wedge a[k] = v \wedge (\forall i. i \neq l \rightarrow a[i] = b[i])$$

Step 3:

$$\begin{aligned} F3 : \quad & a'[k] = b[k] \wedge b[k] \neq v \wedge a[k] = v \wedge (\forall i. i \neq l \rightarrow a[i] = b[i]) \\ & \wedge a'[l] = v \wedge (\forall j. j \neq l \rightarrow a[j] = a'[j]). \end{aligned}$$

Step 4: $\mathcal{I} = \{\lambda, k, l, l-1, l+1\}$

Step 5: we replace universal quantification as follows:

$$\begin{aligned} F5 : \quad & a'[k] = b[k] \wedge b[k] \neq v \wedge a[k] = v \wedge \bigwedge_{i \in \mathcal{I}} (i \neq l \rightarrow a[i] = b[i]) \\ & \wedge a'[l] = v \wedge \bigwedge_{i \in \mathcal{I}} (j \neq l \rightarrow a[j] = a'[j]). \end{aligned}$$

Example

Consider the array property formula

$$F : write(a, l, v)[k] = b[k] \wedge b[k] \neq v \wedge a[k] = v \wedge (\forall i . i \neq l \rightarrow a[i] = b[i])$$

$$\mathcal{I} = \{\lambda, k, l, l-1, l+1\}$$

Step 5 (continued) Expanding produces:

$$\begin{aligned}
F5' : \quad & a'[k] = b[k] \wedge b[k] \neq v \wedge a[k] = v \wedge \\
& (\lambda \neq l \rightarrow a[\lambda] = b[\lambda]) \wedge (k \neq l \rightarrow a[k] = b[k]) \wedge (l \neq l \rightarrow a[l] = b[l]) \wedge \\
& (l-1 \neq l \rightarrow a[l-1] = b[l-1]) \wedge (l+1 \neq l \rightarrow a[l+1] = b[l+1]) \wedge \\
& a'[l] = v \wedge (\lambda \neq l \rightarrow a[\lambda] = a'[\lambda]) \wedge (k \neq l \rightarrow a[k] = a'[k]) \wedge \\
& (l \neq l \rightarrow a[l] = a'[l]) \wedge (l-1 \neq l \rightarrow a[l-1] = a'[l-1]) \wedge \\
& (l+1 \neq l \rightarrow a[l+1] = a'[l+1]).
\end{aligned}$$

Example

Consider the array property formula

$$F : write(a, l, v)[k] = b[k] \wedge b[k] \neq v \wedge a[k] = v \wedge (\forall i. i \neq l \rightarrow a[i] = b[i])$$

$$\mathcal{I} = \{\lambda\} \cup \{k\} \cup \{l, l-1, l+1\} = \{\lambda, k, l, l-1, l+1\}$$

Step 5 (continued): Simplifying produces

$$\begin{aligned} F''5 : \quad & a'[k] = b[k] \wedge b[k] \neq v \wedge a[k] = v \wedge (\lambda \neq l \rightarrow a[\lambda] = b[\lambda]) \\ & \wedge (k \neq l \rightarrow a[k] = b[k]) \wedge a[l-1] = b[l-1] \wedge a[l+1] = b[l+1] \\ & \wedge a'[l] = v \wedge (\lambda \neq l \rightarrow a[\lambda] = a'[\lambda]) \\ & \wedge (k \neq l \rightarrow a[k] = a'[k]) \wedge a[l-1] = a'[l-1] \wedge a[l+1] = a'[l+1]. \end{aligned}$$

Example

Consider the array property formula

$$F : write(a, l, v)[k] = b[k] \wedge b[k] \neq v \wedge a[k] = v \wedge (\forall i. i \neq l \rightarrow a[i] = b[i])$$

Step 6 distinguishes λ from other members of I:

$$\begin{aligned} F6 : \quad & a'[k] = b[k] \wedge b[k] \neq v \wedge a[k] = v \wedge (\lambda \neq l \rightarrow a[\lambda] = b[\lambda]) \\ & \wedge (k \neq l \rightarrow a[k] = b[k]) \wedge a[l-1] = b[l-1] \wedge a[l+1] = b[l+1] \\ & \wedge a'[l] = v \wedge (\lambda \neq l \rightarrow a[\lambda] = a'[\lambda]) \\ & \wedge (k \neq l \rightarrow a[k] = a'[k]) \wedge a[l-1] = a'[l-1] \wedge a[l+1] = a'[l+1] \\ & \wedge \lambda \neq k \wedge \lambda \neq l \wedge \lambda \neq l-1 \wedge \lambda \neq l+1. \end{aligned}$$

Example

Consider the array property formula

$$F : write(a, l, v)[k] = b[k] \wedge b[k] \neq v \wedge a[k] = v \wedge (\forall i . i \neq l \rightarrow a[i] = b[i])$$

Step 6 Simplifying, we have

$$\begin{aligned}
F'6 : \quad & a'[k] = b[k] \wedge b[k] \neq v \wedge a[k] = v \wedge a[\lambda] = b[\lambda] \\
& \wedge a[k] = b[k] \wedge a[l-1] = b[l-1] \wedge a[l+1] = b[l+1] \\
& \wedge a'[l] = v \wedge a[\lambda] = a'[\lambda] \\
& \wedge (k \neq l \rightarrow a[k] = a'[k]) \wedge a[l-1] = a'[l-1] \wedge a[l+1] = a'[l+1] \\
& \wedge \lambda \neq k \wedge \lambda \neq l \wedge \lambda \neq l-1 \wedge \lambda \neq l+1.
\end{aligned}$$

We can use for instance DPLL(T).
Alternative: Case distinction. There are two cases to consider.

(1) If $k{=}l$, then $a'[l]{=}v$ and $a'[k]{=}b[k]$ imply $b[k]{=}v$, yet $b[k]{\neq}v$.
(2) If $k{\neq}l$, then $a[k]{=}v$ and $a[k]{=}b[k]$ imply $b[k]{=}v$, but again $b[k]{\neq}v$.

Hence, F'6 is TA-unsatisfiable, indicating that F is TA-unsatisfiable.

Soundness and Completeness

Theorem (Soundness and Completeness)

Consider a formula F from the array property fragment . The output F6 of Step 6 is T_{arrays}-equisatisfiable to F.

Proof

(Soundness) Step 1-6 preserve satisfiability

(Fi is a logical consequence of F$i-1$).

Soundness and Completeness

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Proof (Completeness)

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Assume that $F6$ is satisfiabile. Clearly F5 has a model.

Soundness and Completeness

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Assume that $F5$ is satisfiabile. Let $\mathcal{A} = (\mathbb{Z}, \text{Elem}, \{a_A\}_{a \in Arrays}, \ldots)$ be a model for F5. Construct a model $\mathcal{B}$ for $F4$ as follows.

For $x \in \mathbb{Z}$: $l(x)$ $(u(x))$ closest left (right) neighbor of x in $\mathcal{I}$.

$$a_{\mathcal{B}}(x) = \begin{cases} a_{\mathcal{A}}(l(x)) & \textit{if } x - l(x) \leq u(x) - x \text{ or } u(x) = \infty \\ a_{\mathcal{A}}(u(x)) & \textit{if } x - l(x) > u(x) - x \text{ or } l(x) = -\infty \end{cases}$$

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If F3 has model then F2 has model

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Given a formula F containing an occurrence of a write term $write(a, i, v)$, we can substitute every occurrence of $write(a, i, v)$ with a fresh variable a' and explan the relationship between a' and a.

If F2 has a model then F1 has a model.

Step 1: Put F in NNF: NNF F1 is equivalent to F.

Theories of arrays

Theorem (Complexity) Suppose $(T_{index} \cup T_{elem})$-satisfiability is in NP. For sub-fragments of the array property fragment in which formulae have bounded-size blocks of quantifiers, T_{arrays}-satisfiability is NP-complete.

Proof NP-hardness is clear.

That the problem is in NP follows easily from the procedure: instantiating a block of n universal quantifiers quantifying subformula G over index set I produces $|I| \cdot n$ new subformulae, each of length polynomial in the length of G. Hence, the output of Step 6 is of length only a polynomial factor greater than the input to the procedure for fixed n.

Program verification

Example: Does BubbleSort return a sorted array?

$$
\begin{array}{l}
\text{int } [\,]\ \textsc{BubbleSort}(\text{int}[\,]\ a)\ \{ \\
\quad \text{int } i, j, t; \\
\quad \text{for } (i := |a| - 1; i > 0; i := i - 1)\ \{ \\
\qquad \text{for } (j := 0; j < i; j := j + 1)\ \{ \\
\qquad\quad \text{if } (a[j] > a[j+1])\{t := a[j]; \\
\qquad\qquad\qquad\qquad\qquad\qquad a[j] := a[j+1]; \\
\qquad\qquad\qquad\qquad\qquad\qquad a[j+1] := t\}; \\
\}\}\ \text{return } a\}
\end{array}
$$

Program Verification

$-1 \leq i < |a| \wedge$
partitioned$(a, 0, i, i+1, |a|-1) \wedge$
sorted$(a, i, |a|-1)$

$-1 \leq i < |a| \wedge 0 \leq j \leq i \wedge$
partitioned$(a, 0, i, i+1, |a|-1) \wedge$
sorted$(a, i, |a|-1)$
partitioned$(a, 0, j-1, j, j)$ $\quad C_2$

Example: Does BubbleSort return a sorted array?

```
int [] BubbleSort(int[] a) {
  int i, j, t;
  for (i := |a| − 1; i > 0; i := i − 1) {
    for (j := 0; j < i; j := j + 1) {
      if (a[j] > a[j + 1]){t := a[j];
                            a[j] := a[j + 1];
                            a[j + 1] := t};
}} return a}
```

Generate verification conditions and prove that they are valid

Predicates:

- sorted(a, l, u): $\quad \forall i, j (l \leq i \leq j \leq u \rightarrow a[i] \leq a[j])$
- partitioned(a, l_1, u_1, l_2, u_2): $\quad \forall i, j (l_1 \leq i \leq u_1 \leq l_2 \leq j \leq u_2 \rightarrow a[i] \leq a[j])$

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- sorted(a, l, u): $\forall i, j (l \leq i \leq j \leq u \rightarrow a[i] \leq a[j])$
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To prove: $C_2(a) \wedge \text{Update}(a, a') \rightarrow C_2(a')$

Another Situation

Insertion of an element c in a sorted array a of length n

for $(i := 1; i \leq n; i := i + 1)$ {
    if $a[i] \geq c\{n := n + 1$
        for $(j := n; j > i; j := j - 1)\{a[i] := a[i - 1]\}$
        $a[i] := c$; return a
}} $a[n + 1] := c$; return a

Task:

If the array was sorted before insertion it is sorted also after insertion.

$\text{Sorted}(a, n) \wedge \text{Update}(a, n, a', n') \wedge \neg\text{Sorted}(a', n') \models_{\mathcal{T}} \perp$?

Another Situation

Task:

If the array was sorted before insertion it is sorted also after insertion.

$\text{Sorted}(a, n) \wedge \text{Update}(a, n, a', n') \wedge \neg\text{Sorted}(a', n') \models_{\mathcal{T}} \perp$?

$\text{Sorted}(a, n)$	$\forall i, j(1 \leq i \leq j \leq n \rightarrow a[i] \leq a[j])$
$\text{Update}(a, n, a', n')$	$\forall i((1 \leq i \leq n \wedge a[i] < c) \rightarrow a'[i] = a[i])$
	$\forall i((c \leq a(1) \rightarrow a'[1] := c)$
	$\forall i((a[n] < c \rightarrow a'[n+1] := c)$
	$\forall i((1 \leq i-1 \leq i \leq n \wedge a[i-1] < c \wedge a[i] \geq c) \rightarrow (a'[i] = c)$
	$\forall i((1 \leq i-1 \leq i \leq n \wedge a[i-1] \geq c \wedge a[i] \geq c \rightarrow a'[i] := a[i-1])$
	$n' := n+1$
$\neg\text{Sorted}(a', n')$	$\exists k, l(1 \leq k \leq l \leq n' \wedge a'k] > a'[l])$

Beyond the array property fragment

Extension: New arrays defined by case distinction – $\mathrm{Def}(f')$

$$\forall \overline{x}(\phi_i(\overline{x}) \rightarrow f'(\overline{x}){=}s_i(\overline{x})) \qquad i \in I, \text{ where } \phi_i(\overline{x}) \wedge \phi_j(\overline{x}) \models_{\mathcal{T}_0} \perp \text{ for } i{\neq}j \quad (1)$$

$$\forall \overline{x}(\phi_i(\overline{x}) \rightarrow t_i(\overline{x}){\leq}f'(\overline{x}){\leq}s_i(\overline{x})) \qquad i \in I, \text{ where } \phi_i(\overline{x}) \wedge \phi_j(\overline{x}) \models_{\mathcal{T}_0} \perp \text{ for } i{\neq}j \quad (2)$$

where s_i, t_i are terms over the signature Σ such that $\mathcal{T}_0 \models \forall \overline{x}(\phi_i(\overline{x}){\rightarrow}t_i(\overline{x}){\leq}s_i(\overline{x}))$ for all $i \in I$.

$\mathcal{T}_0 \subseteq \mathcal{T}_0 \wedge \mathrm{Def}(f')$ has the property that for every set G of ground clauses in which there are no nested applications of f':

$$\mathcal{T}_0 \wedge \mathrm{Def}(f') \wedge G \models \perp \quad \text{iff} \quad \mathcal{T}_0 \wedge \mathrm{Def}(f')[G] \wedge G$$

(sufficient to use instances of axioms in $\mathrm{Def}(f')$ which are relevant for G)

- Some of the syntactic restrictions of the array property fragment can be lifted

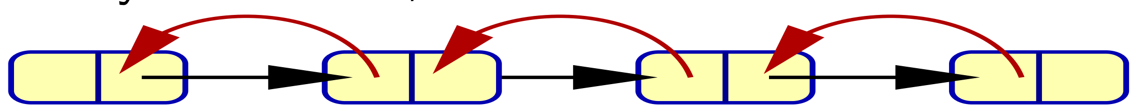
Pointer Structures

[McPeak, Necula 2005]

- pointer sort p, scalar sort s; pointer fields (p $\rightarrow$ p); scalar fields (p $\rightarrow$ s);
- axioms: $\forall p \;\; \mathcal{E} \vee \mathcal{C}$; $\quad$ $\mathcal{E}$ contains disjunctions of pointer equalities
  $\mathcal{C}$ contains scalar constraints

Assumption: If $f_1(f_2(\dots f_n(p)))$ occurs in axiom, the axiom also contains:
$$p{=}\text{null} \vee f_n(p){=}\text{null} \vee \cdots \vee f_2(\dots f_n(p))){=}\text{null}$$

Example: doubly-linked lists; ordered elements



$$\forall p \; (p \neq \text{null} \wedge p.\text{next} \neq \text{null} \;\; \rightarrow \;\; p.\text{next.prev} = p)$$
$$\forall p \; (p \neq \text{null} \wedge p.\text{prev} \neq \text{null} \;\; \rightarrow \;\; p.\text{prev.next} = p)$$
$$\forall p \; (p \neq \text{null} \wedge p.\text{next} \neq \text{null} \;\; \rightarrow \;\; p.\text{info} \leq p.\text{next.info})$$

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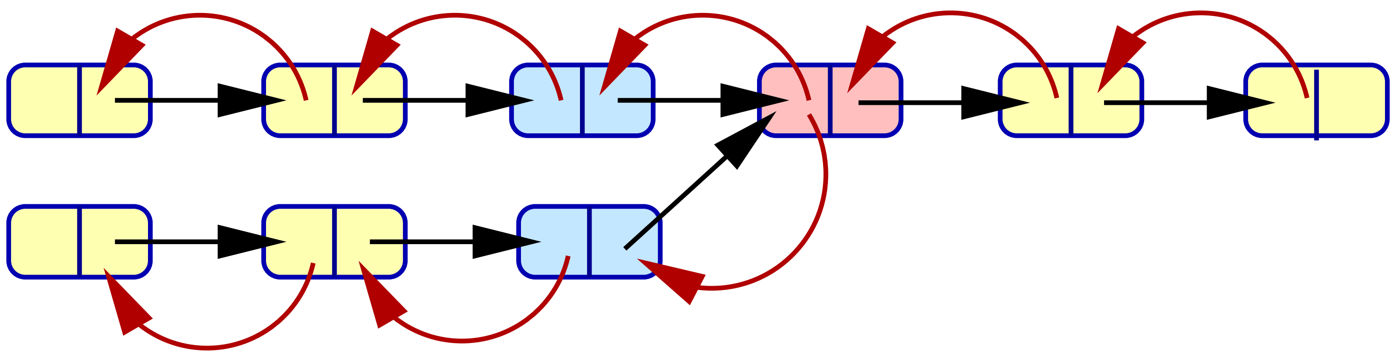
$p{=}\text{null} \vee f_n(p){=}\text{null} \vee \cdots \vee f_2(\ldots f_n(p))){=}\text{null}$

Theorem. K set of clauses in the fragment above. Then for every set G of ground clauses, $(K \cup G) \cup \mathcal{T}_s \models \perp$ iff $K^{[G]} \cup \mathcal{T}_s \models \perp$

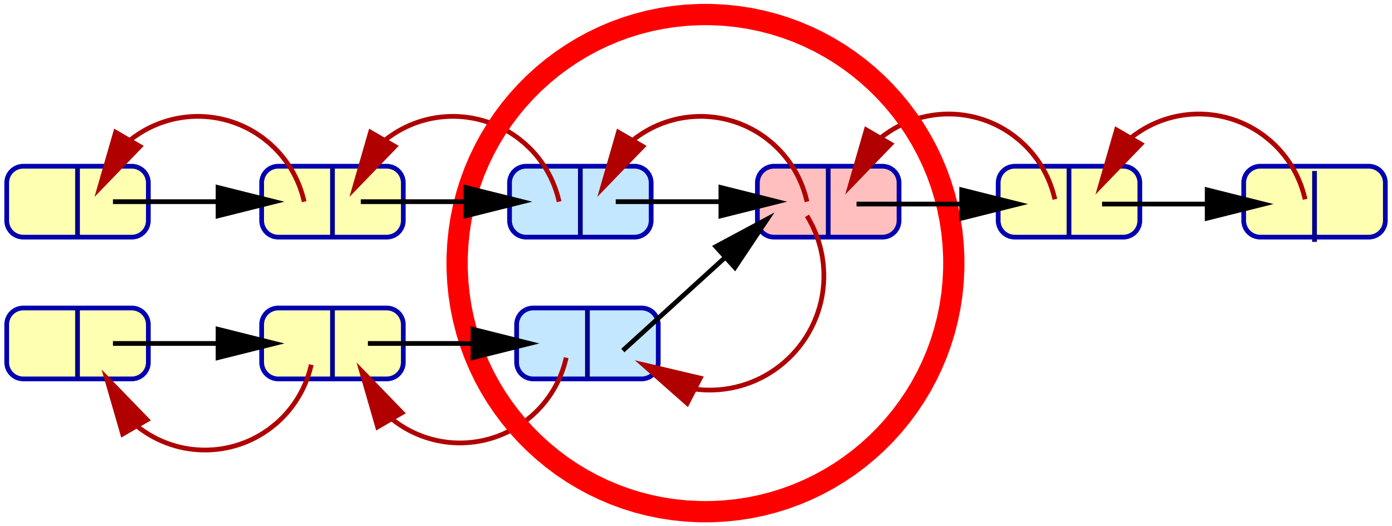
where $K^{[G]}$ is the set of instances of K in which the variables are replaced by subterms in G.

Example: A theory of doubly-linked lists



$$\forall p\ (p \neq \text{null} \wedge p.\text{next} \neq \text{null} \ \rightarrow\ p.\text{next}.\text{prev} = p)$$
$$\forall p\ (p \neq \text{null} \wedge p.\text{prev} \neq \text{null} \ \rightarrow\ p.\text{prev}.\text{next} = p)$$
$$\wedge\ \ c{\neq}\text{null} \wedge c.\text{next}{\neq}\text{null} \wedge d{\neq}\text{null} \wedge d.\text{next}{\neq}\text{null} \wedge c.\text{next}{=}d.\text{next} \wedge c \neq d \quad \models \quad \perp$$

Example: A theory of doubly-linked lists

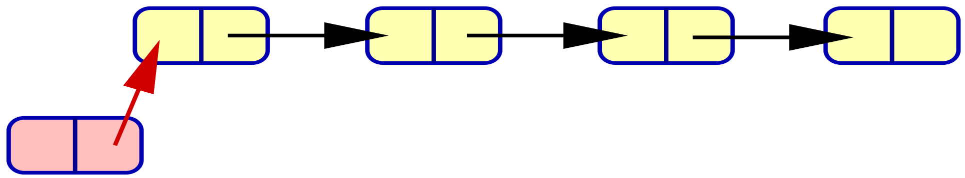


$(c{\neq}\mathrm{null} \wedge c.\mathrm{next}{\neq}\mathrm{null} \rightarrow c.\mathrm{next.prev}{=}c)$ $\quad$ $(c.\mathrm{next}{\neq}\mathrm{null} \wedge c.\mathrm{next.next}{\neq}\mathrm{null} \rightarrow c.\mathrm{next.next.prev}{=}c.\mathrm{nex}$

$(d{\neq}\mathrm{null} \wedge d.\mathrm{next}{\neq}\mathrm{null}{\rightarrow}d.\mathrm{next.prev}{=}d)$ $\quad$ $(d.\mathrm{next}{\neq}\mathrm{null} \wedge d.\mathrm{next.next}{\neq}\mathrm{null}{\rightarrow}d.\mathrm{next.next.prev}{=}d.\mathrm{nex}$

$$\wedge \quad c{\neq}\mathrm{null} \wedge c.\mathrm{next}{\neq}\mathrm{null} \wedge d{\neq}\mathrm{null} \wedge d.\mathrm{next}{\neq}\mathrm{null} \wedge c.\mathrm{next}{=}d.\mathrm{next} \wedge c \neq d \quad \models \quad \perp$$

Example: List insertion



Initially list is sorted: $p.\mathrm{next} \neq \mathrm{null} \rightarrow p.\mathrm{prio} \geq p.\mathrm{next}.\mathrm{prio}$

$c.\mathrm{prio} = x,\ c.\mathrm{next} = \mathrm{null}$

for all $p \neq c$ **do**

if $p.\mathrm{prio} \leq x$ **then if** $\mathrm{First}(p)$ **then** $c.\mathrm{next}' = p,\ \mathrm{First}'(c),\ \neg\mathrm{First}'(p)$ **endif**; $p.\mathrm{next}' = p.\mathrm{next}$

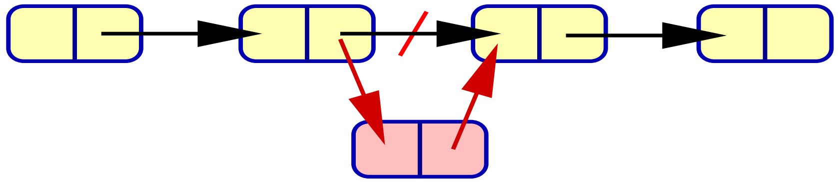
$p.\mathrm{prio} > x$ **then** **case** $p.\mathrm{next} = \mathrm{null}$ **then** $p.\mathrm{next}' := c,\ c.\mathrm{next}' = \mathrm{null}$

$p.\mathrm{next} \neq \mathrm{null} \wedge p.\mathrm{next}.\mathrm{prio} > x$ **then** $p.\mathrm{next}' = p.\mathrm{next}$

$p.\mathrm{next} \neq \mathrm{null} \wedge p.\mathrm{next}.\mathrm{prio} \leq x$ **then** $p.\mathrm{next}' = c,\ c.\mathrm{next}' = p.\mathrm{nex}$

Verification task: After insertion list remains sorted

Example: List insertion



Initially list is sorted: $p.\text{next} \neq \text{null} \rightarrow p.\text{prio} \geq p.\text{next.prio}$

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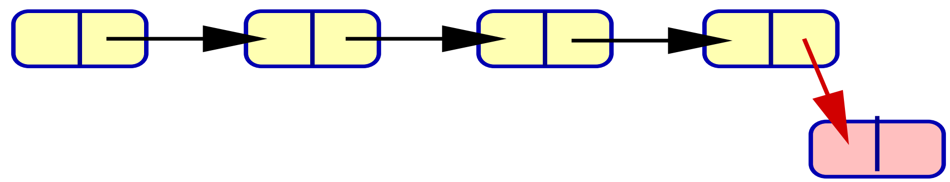
Verification task: After insertion list remains sorted

Example: List insertion



Initially list is sorted: $p.\text{next} \neq \text{null} \rightarrow p.\text{prio} \geq p.\text{next.prio}$

$c.\text{prio} = x,\ c.\text{next} = \text{null}$

for all $p \neq c$ **do**

if $p.\text{prio} \leq x$ **then if** $\text{First}(p)$ **then** $c.\text{next}' = p,\ \text{First}'(c),\ \neg\text{First}'(p)$ **endif**; $p.\text{next}' = p.\text{next}$

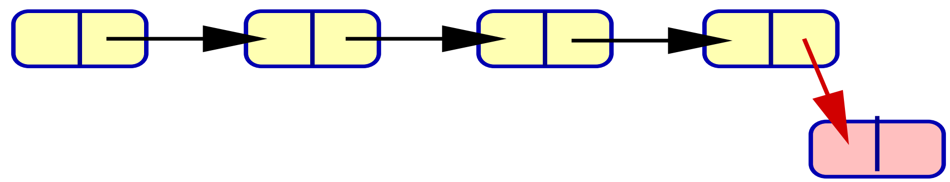
$p.\text{prio} > x$ **then** **case** $p.\text{next} = \text{null}$ **then** $p.\text{next}' := c,\ c.\text{next}' = \text{null}$

$p.\text{next} \neq \text{null} \wedge p.\text{next.prio} > x$ **then** $p.\text{next}' = p.\text{next}$

$p.\text{next} \neq \text{null} \wedge p.\text{next.prio} \leq x$ **then** $p.\text{next}' = c,\ c.\text{next}' = p.\text{nex}$

Verification task: After insertion list remains sorted

Example: List insertion



Initially list is sorted: $\forall p(p.\text{next} \neq \text{null} \rightarrow p.\text{prio} \geq p.\text{next}.\text{prio})$

$\forall p(p{\neq}\text{null} \wedge p{\neq}c \wedge \text{prio}(p){\leq}x \wedge \text{First}(p) \rightarrow \text{next}'(c){=}p \wedge \text{First}'(c))$

$\forall p(p{\neq}\text{null} \wedge p{\neq}c \wedge \text{prio}(p){\leq}x \wedge \text{First}(p) \rightarrow \text{next}'(p){=}\text{next}(p) \wedge \neg\text{First}'(p))$

$\forall p(p{\neq}\text{null} \wedge p{\neq}c \wedge \text{prio}(p){\leq}x \wedge \neg\text{First}(p) \rightarrow \text{next}'(p){=}\text{next}(p))$

$\forall p(p{\neq}\text{null} \wedge p{\neq}c \wedge \text{prio}(p){>}x \wedge \text{next}(p){=}\text{null} \rightarrow \text{next}'(p){=}c$

$\forall p(p{\neq}\text{null} \wedge p{\neq}c \wedge \text{prio}(p){>}x \wedge \text{next}(p){=}\text{null} \rightarrow \text{next}'(c){=}\text{null})$

$\forall p(p{\neq}\text{null} \wedge p{\neq}c \wedge \text{prio}(p){>}x \wedge \text{next}(p){\neq}\text{null} \wedge \text{prio}(\text{next}(p)){>}x \rightarrow \text{next}'(p){=}\text{next}(p))$

$\forall p(p{\neq}\text{null} \wedge p$ … $(p){=}c$

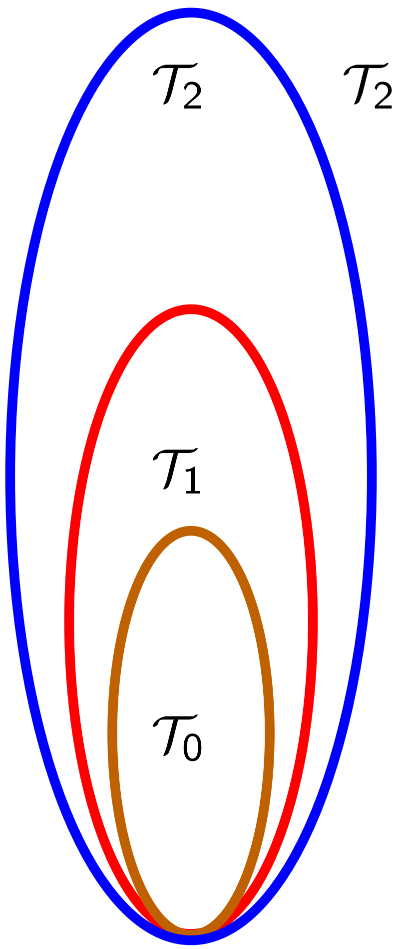
$\forall p(p{\neq}\text{null} \wedge p$ … $(c){=}\text{next}(p))$

We only need to use instances in which variables are replaced by ground subterms occurring in the problem

---

To check: $\text{Sorted}(\text{next}, \text{prio}) \wedge \text{Update}(\text{next}, \text{next}') \wedge p_0.\text{next}'{\neq}\text{null}{\wedge}p_0.\text{prio}{\not\geq}p_0.\text{next}'.\text{prio} \models \perp$

Example: List insertion



$$\mathcal{T}_2 = \mathcal{T}_1 \cup \mathsf{Update}(\mathsf{next}, \mathsf{next'})$$

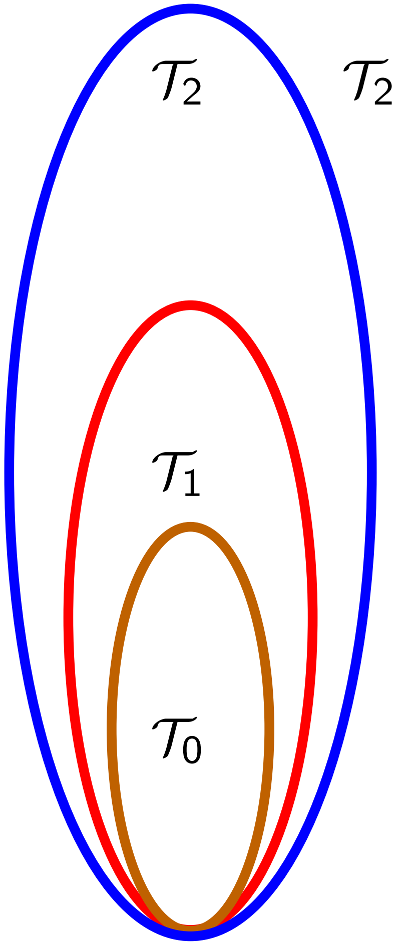
$$\mathcal{T}_1 = \mathcal{T}_0 \cup \mathsf{Sorted}(\mathsf{next})$$

$$\mathcal{T}_0 = (\mathsf{Lists}, \mathsf{next})$$

To show:

$$\mathcal{T}_2 \cup \underbrace{\neg \mathsf{Sorted}(\mathsf{next'})}_{G} \models \perp$$

Example: List insertion



To show:

$\mathcal{T}_2 = \mathcal{T}_1 \cup \boxed{\text{Update}(\text{next}, \text{next}')}$

$\mathcal{T}_2 \cup \underbrace{\neg\text{Sorted}(\text{next}')}_{G} \models \perp$

Instantiate:

Hierarchical reasoning:

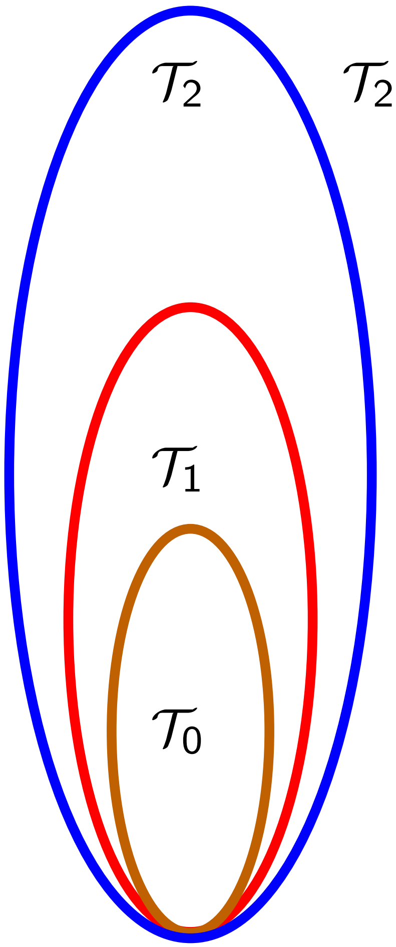
$\mathcal{T}_1 \cup \underbrace{\boxed{\text{Update}(\text{next}, \text{next}')[G]} \cup G}_{G'} \models$

$\mathcal{T}_1 = \mathcal{T}_0 \cup \text{Sorted}(\text{next})$

$\mathcal{T}_1 \cup G'(\text{next}) \models \perp$

$\mathcal{T}_0 = (\text{Lists}, \text{next})$

Example: List insertion



To show:

$$\mathcal{T}_2 = \mathcal{T}_1 \cup \mathsf{Update}(\mathsf{next}, \mathsf{next}')$$

$$\mathcal{T}_2 \cup \underbrace{\neg \mathsf{Sorted}(\mathsf{next}')}_{G} \models \perp$$

$$\Downarrow$$

$$\mathcal{T}_1 = \mathcal{T}_0 \cup \mathsf{Sorted}(\mathsf{next})$$

$$\mathcal{T}_1 \cup G'(\mathsf{next}) \models \perp$$

$$\Downarrow$$

$$\mathcal{T}_0 = (\mathsf{Lists}, \mathsf{next})$$

$$\mathcal{T}_0 \cup G'' \models \perp$$

More general concept

Local Theory Extensions

Satisfiability of formulae with quantifiers

Goal: generalize the ideas for extensions of theories

Example: Strict monotonicity

$$\mathbb{R} \cup \mathbb{Z} \cup \mathrm{Mon}(f) \cup \underbrace{(a < b \wedge f(a) = f(b) + 1)}_{G} \models \perp$$

$$\mathrm{Mon}(f) \qquad \forall i, j (i < j \rightarrow f(i) < f(j))$$

Problems:

- A prover for $\mathbb{R} \cup \mathbb{Z}$ does not know about f
- A prover for first-order logic may have problems with the reals and integers
- DPLL(T) cannot be used (Mon, $\mathbb{Z}, \mathbb{R}$: non-disjoint signatures)
- SMT provers may have problems with the universal quantifiers

Our goal: reduce search: consider certain instances $\mathrm{Mon}(f)[G]$ without loss of completeness

hierarchical/modular reasoning:
reduce to checking satisfiability of a set of constraints over $\mathbb{R} \cup \mathbb{Z}$

Local theory extensions

Solution: Local theory extensions

$\mathcal{K}$ set of equational clauses; $\mathcal{T}_0$ theory; $\mathcal{T}_1 = \mathcal{T}_0 \cup \mathcal{K}$

(Loc) $\mathcal{T}_0 \subseteq \mathcal{T}_1$ is local, if for ground clauses G,
$\mathcal{T}_0 \cup \mathcal{K} \cup G \models \perp$ iff $\mathcal{T}_0 \cup \mathcal{K}[G] \cup G$ has no (partial) model

Various notions of locality, depending of the instances to be considered:
stable locality, order locality; extended locality.

Example: Strict monotonicity

$$\boxed{\mathbb{R} \cup \mathbb{Z} \cup \mathrm{Mon}(f) \cup \underbrace{(a < b \wedge f(a) = f(b) + 1)}_{G} \models \perp}$$

Base theory ($\mathbb{R} \cup \mathbb{Z}$)	Extension
$a < b$	$f(a) = f(b) + 1$
	$\forall i, j (i < j \rightarrow f(i) < f(j))$

Example: Strict monotonicity

$$\mathbb{R} \cup \mathbb{Z} \cup \mathrm{Mon}(f) \cup \underbrace{(a < b \wedge f(a) = f(b) + 1)}_{G} \models \perp$$

Extension is local $\mapsto$ replace axiom with ground instances

Base theory ($\mathbb{R} \cup \mathbb{Z}$)	Extension
$a < b$	$f(a) = f(b) + 1$
	$a < b \rightarrow f(a) < f(b)$
	$b < a \rightarrow f(b) < f(a)$

Solution 1:

$SMT(\mathbb{R} \cup \mathbb{Z} \cup UIF)$

Example: Strict monotonicity

$$\mathbb{R} \cup \mathbb{Z} \cup \mathrm{Mon}(f) \cup \underbrace{(a < b \wedge f(a) = f(b) + 1)}_{G} \models \perp$$

Extension is local $\mapsto$ replace axiom with ground instances

Add congruence axioms. Replace pos-terms with new constants

Base theory ($\mathbb{R} \cup \mathbb{Z}$)	Extension
$a < b$	$f(a) = f(b) + 1$
	$a < b \rightarrow f(a) < f(b)$
	$b < a \rightarrow f(b) < f(a)$
	$a = b \rightarrow f(a) = f(b)$

Solution 2:

Hierarchical reasoning

Example: Strict monotonicity

$$\mathbb{R} \cup \mathbb{Z} \cup \mathsf{Mon}(f) \cup \underbrace{(a < b \wedge f(a) = f(b) + 1)}_{G} \models \perp$$

Extension is local $\mapsto$ replace axiom with ground instances

Replace f-terms with new constants

Add definitions for the new constants

Base theory ($\mathbb{R} \cup \mathbb{Z}$)	Extension
$a < b$	$a_1 = b_1 + 1$
	$a < b \rightarrow a_1 < b_1$
	$b < a \rightarrow b_1 < a_1$
	$a = b \rightarrow a_1 = b_1$

Example: Strict monotonicity

$$\mathbb{R} \cup \mathbb{Z} \cup \mathsf{Mon}(f) \cup \underbrace{(a < b \wedge f(a) = f(b) + 1)}_{G} \models \perp$$

Extension is local $\mapsto$ replace axiom with ground instances

Replace f-terms with new constants

Add definitions for the new constants

Base theory ($\mathbb{R} \cup \mathbb{Z}$)	Extension
$a < b$	$a_1 = f(a)$
$a_1 = b_1 + 1$	$b_1 = f(b)$
$a < b \rightarrow a_1 < b_1$	
$b < a \rightarrow b_1 < a_1$	
$a = b \rightarrow a_1 = b_1$	

Reasoning in local theory extensions

Locality: $\mathcal{T}_0 \cup \mathcal{K} \cup G \models \perp$ iff $\mathcal{T}_0 \cup \mathcal{K}[G] \cup G \models \perp$

Problem: Decide whether $\mathcal{T}_0 \cup \mathcal{K}[G] \cup G \models \perp$

Solution 1: Use $SMT(\mathcal{T}_0{+}UIF)$: possible only if $\mathcal{K}[G]$ ground

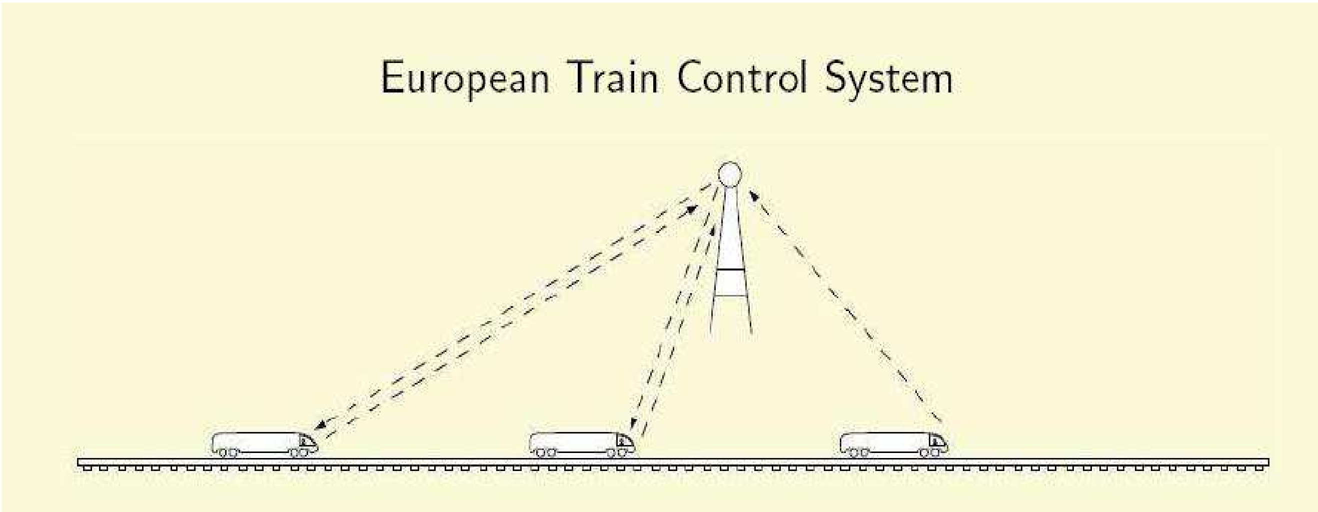
Solution 2: Hierarchic reasoning [VS'05]

reduce to satisfiability in $\mathcal{T}_0$: applicable in general

$\mapsto$ parameterized complexity

Example

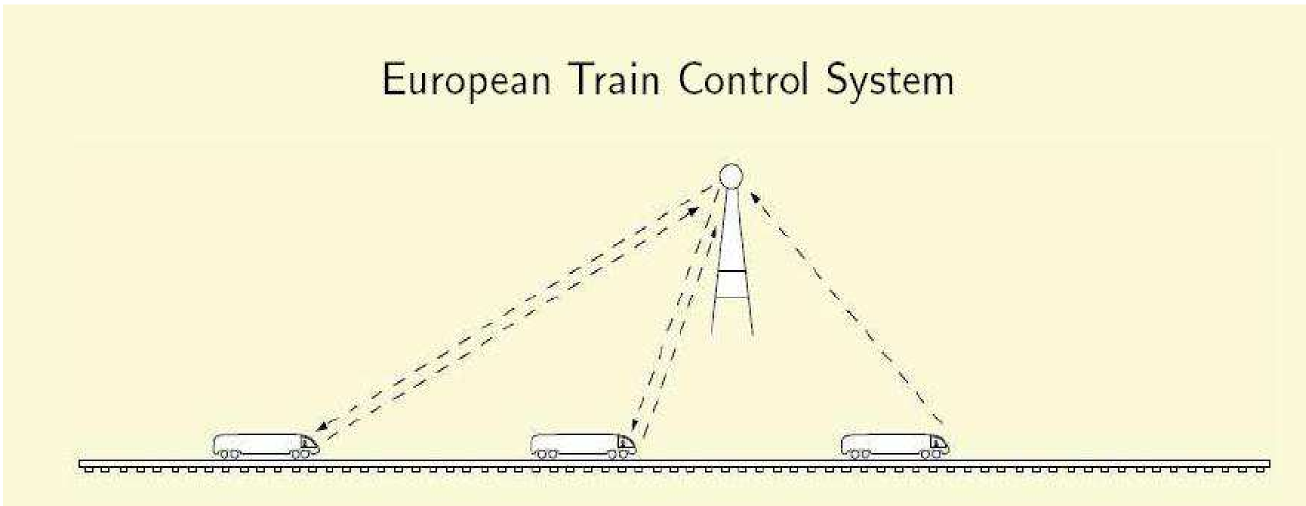
Simplified version of ETCS Case Study [Jacobs,VS'06, Faber,Jacobs,VS'07]



Number of trains:	$n \geq 0$	$\mathbb{Z}$
Minimum and maximum speed of trains:	$0 \leq min < max$	$\mathbb{R}$
Minimum secure distance:	$l_{alarm} > 0$	$\mathbb{R}$
Time between updates:	$\Delta t > 0$	$\mathbb{R}$
Train positions before and after update:	$pos(i), pos'(i)$	$: \mathbb{Z} \rightarrow \mathbb{R}$

Example

Simplified version of ETCS Case Study [Jacobs,VS'06, Faber,Jacobs,VS'07]



Update(pos, pos′) :

- $\forall i \ (i = 0 \rightarrow pos(i) + \Delta t * \min \leq pos'(i) \leq pos(i) + \Delta t * \max)$
- $\forall i \ (0 < i < n \ \wedge \ pos(i-1) > 0 \ \wedge \ pos(i-1) - pos(i) \geq l_{\text{alarm}}$
  $\rightarrow pos(i) + \Delta t * \min \leq pos'(i) \leq pos(i) + \Delta t * \max)$

...

Example

Safety property: No collisions $\quad \text{Safe}(\text{pos}): \ \forall i, j (i < j \rightarrow \text{pos}(i) > \text{pos}(j))$

Inductive invariant: $\text{Safe}(\text{pos}) \wedge \text{Update}(\text{pos}, \text{pos}') \wedge \neg \text{Safe}(\text{pos}') \models_{\mathcal{T}_S} \perp$

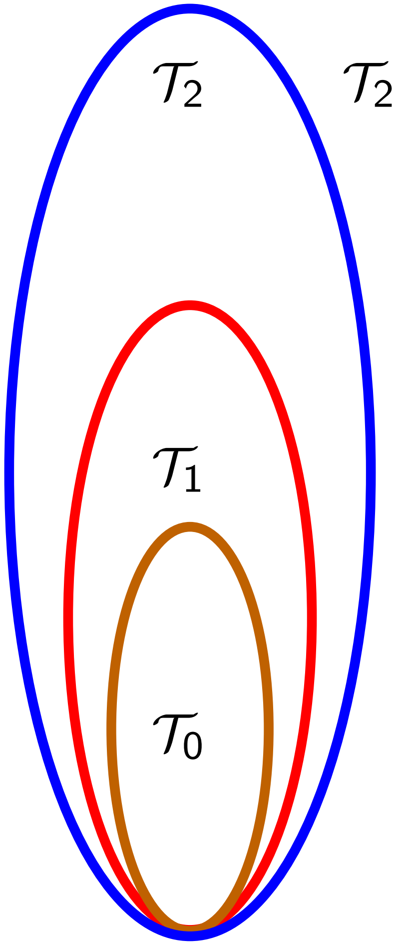
where $\mathcal{T}_S$ is the extension of the (disjoint) combination $\mathbb{R} \cup \mathbb{Z}$
with two functions, $\text{pos}, \text{pos}' : \mathbb{Z} \rightarrow \mathbb{R}$

Our idea: Use chains of "instantiation" + reduction.

Example

To show:

$$\mathcal{T}_2 \cup \underbrace{\neg \mathsf{Safe}(\mathsf{pos}')}_{G} \models \perp$$

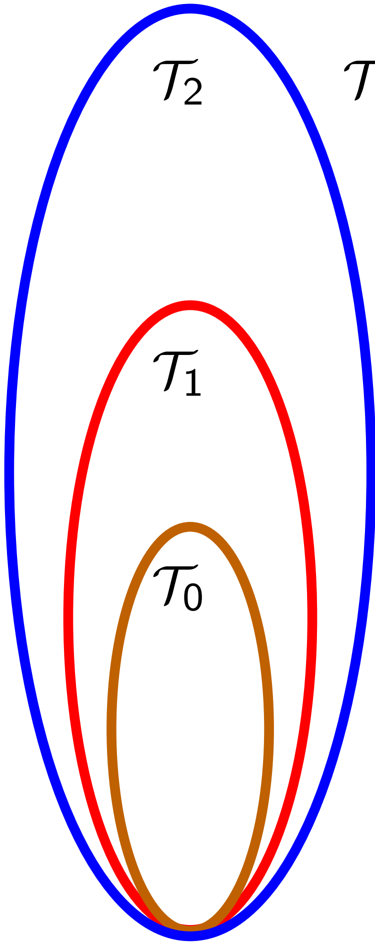


$$\mathcal{T}_2 = \mathcal{T}_1 \cup \mathsf{Update}(\mathsf{pos}, \mathsf{pos}')$$

$$\mathcal{T}_1 = \mathcal{T}_0 \cup \mathsf{Safe}(\mathsf{pos})$$

$$\mathcal{T}_0 = \mathbb{R} \cup \mathbb{Z}$$

Example



To show:

$$\mathcal{T}_2 = \mathcal{T}_1 \cup \mathsf{Update}(\mathsf{pos}, \mathsf{pos}') \qquad \mathcal{T}_2 \cup \underbrace{\neg \mathsf{Safe}(\mathsf{pos}')}_{G} \models \perp$$

$$\Downarrow$$

$$\mathcal{T}_1 = \mathcal{T}_0 \cup \mathsf{Safe}(\mathsf{pos}) \qquad \mathcal{T}_1 \cup G'(\mathsf{pos}) \models \perp$$

$$\Downarrow$$

$$\mathcal{T}_0 = \mathbb{R} \cup \mathbb{Z} \qquad \mathcal{T}_0 \cup G'' \models \perp$$

$$\Phi(c, \overline{c}_{\mathsf{pos}'}, \overline{d}_{\mathsf{pos}}, n, l_{\mathsf{alarm}}, \mathsf{min}, \mathsf{max}, \Delta t) \models \perp$$

Method 1: SAT checking/ Counterexample generation

Method 2: Quantifier elimination

relationships between parameters which guarantee safety

More complex ETCS Case studies

[Faber, Jacobs, VS, 2007]

- Take into account also:
  - Emergency messages
  - Durations

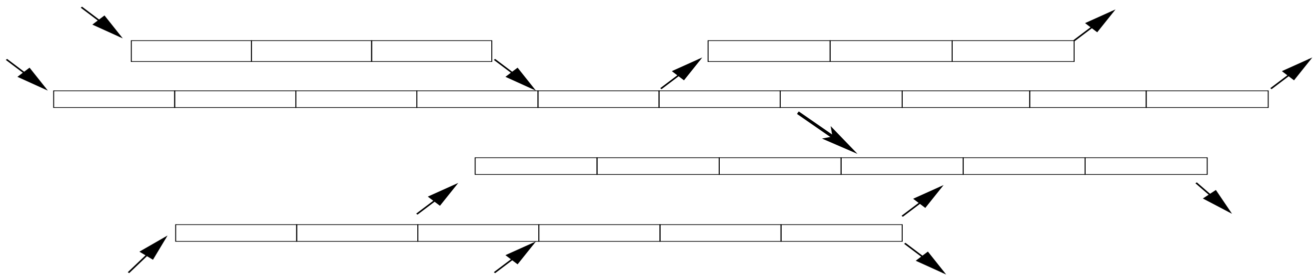
- Specification language: CSP-OZ-DC
  - Reduction to satisfiability in theories for which decision procedures exist

- **Tool chain:** [Faber, Ihlemann, Jacobs, VS]

CSP-OZ-DC $\mapsto$ Transition constr. $\mapsto$ Decision procedures (H-PILoT)

Example 2: Parametric topology

- **Complex track topologies** [Faber, Ihlemann, Jacobs, VS, ongoing work]

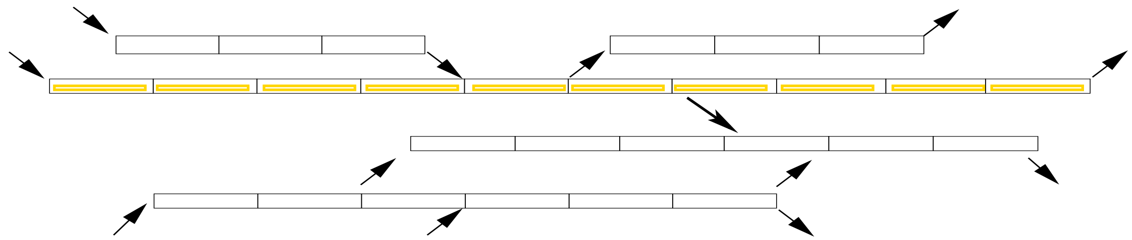


Assumptions:

- No cycles
- in-degree (out-degree) of associated graph at most 2.

Parametricity and modularity

- **Complex track topologies** [Faber, Ihlemann, Jacobs, VS, ongoing work]



Assumptions:

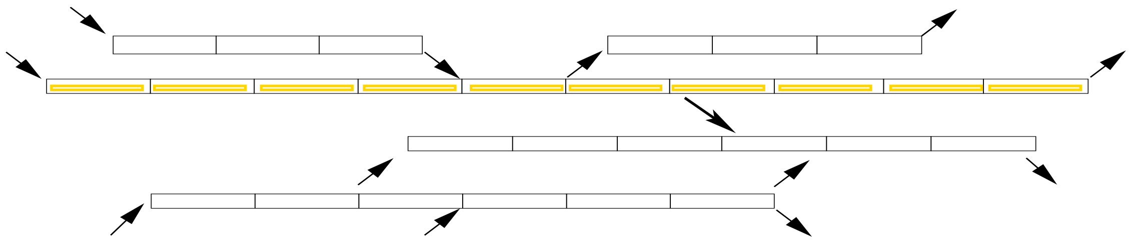
- No cycles
- in-degree (out-degree) of associated graph at most 2.

Approach:

- Decompose the system in trajectories (linear rail tracks; may overlap)
- Task 1: - Prove safety for trajectories with incoming/outgoing trains
  - Conclude that for control rules in which trains have sufficient freedom (and if trains are assigned unique priorities) safety of all trajectories implies safety of the whole system
- Task 2: - General constraints on parameters which guarantee safety

Parametricity and modularity

- **Complex track topologies** [Faber, Ihlemann, Jacobs, VS, ongoing work]



Assumptions:

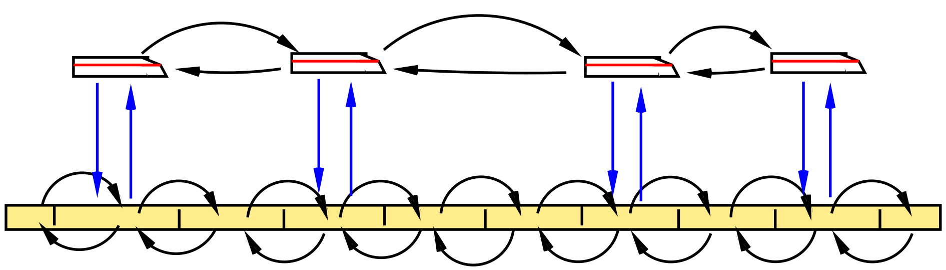
- No cycles
- in-degree (out-degree) of associated graph at most 2.

Data structures:

p_1: trains

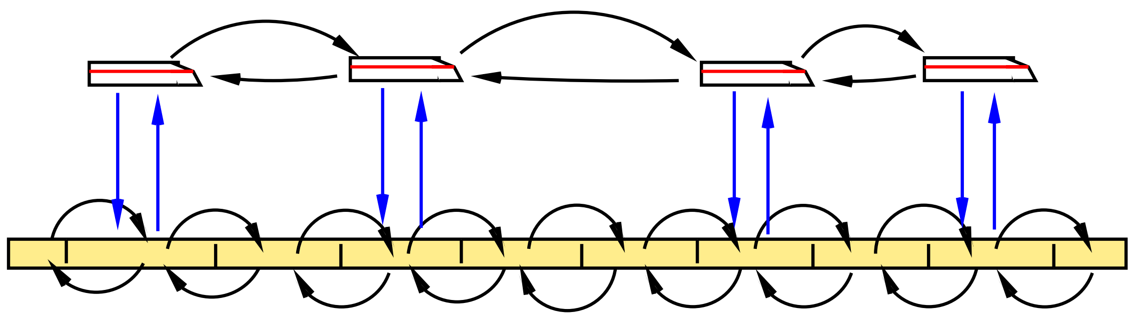
- 2-sorted pointers

p_2: segments



- scalar fields (f:$p_i \rightarrow \mathbb{R}$, g:$p_i \rightarrow \mathbb{Z}$)
- updates efficient decision procedures (H-PiLoT)

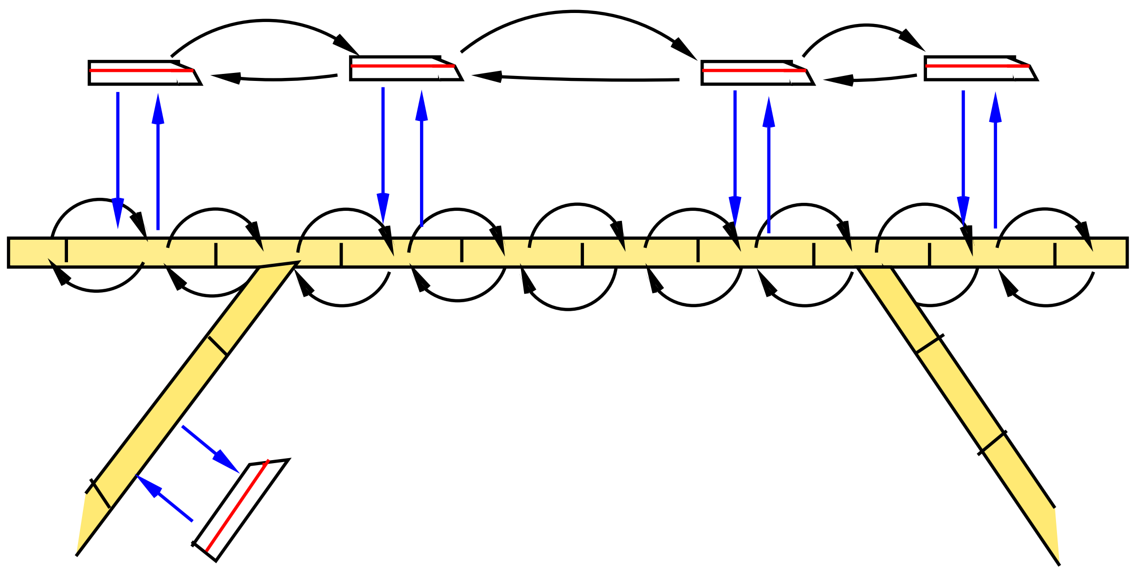
Incoming and outgoing trains



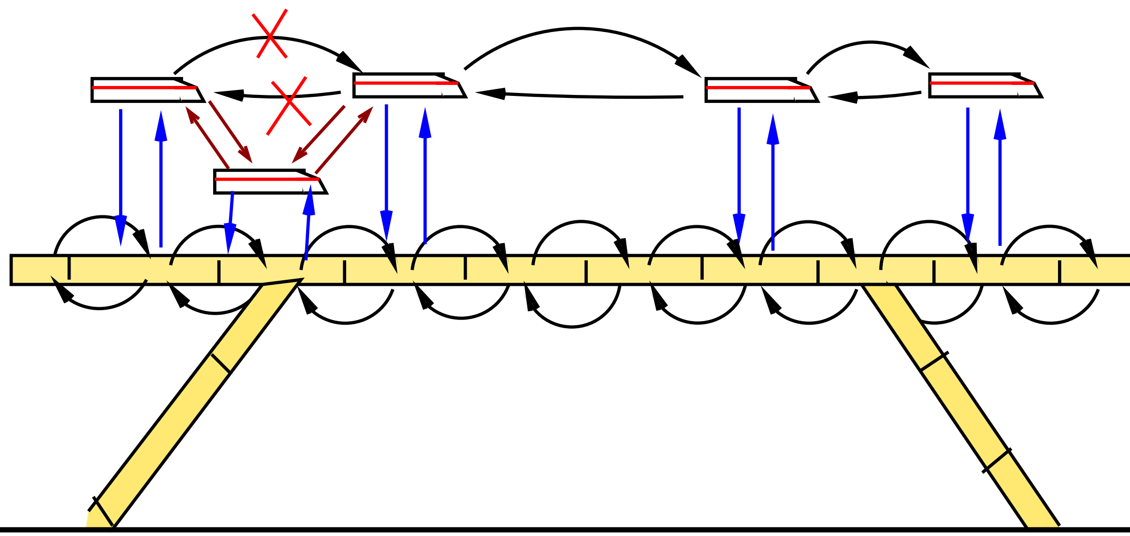
Example 1: Speed Update

$$
\begin{aligned}
&\mathrm{pos}(t) < \mathrm{length}(\mathrm{segm}(t)) - d \;\rightarrow\; 0 \leq \mathrm{spd}'(t) \leq \mathrm{lmax}(\mathrm{segm}(t))\\
&\mathrm{pos}(t) \geq \mathrm{length}(\mathrm{segm}(t)) - d \;\wedge\; \mathrm{alloc}(\mathrm{next}_s(\mathrm{segm}(t))) = \mathrm{tid}(t)\\
&\qquad \rightarrow\; 0 \leq \mathrm{spd}'(t) \leq \min(\mathrm{lmax}(\mathrm{segm}(t)), \mathrm{lmax}(\mathrm{next}_s(\mathrm{segm}(t)))\\
&\mathrm{pos}(t) \geq \mathrm{length}(\mathrm{segm}(t)) - d \;\wedge\; \mathrm{alloc}(\mathrm{next}_s(\mathrm{segm}(t))) \neq \mathrm{tid}(t)\\
&\qquad \rightarrow\; \mathrm{spd}'(t) = \max(\mathrm{spd}(t) - \mathrm{decmax}, 0)
\end{aligned}
$$

Incoming and outgoing trains



Incoming and outgoing trains



Example 2: Enter Update (also updates for segm', spd', pos', train')

Assume: $s_1 \neq \mathrm{null}_s,\ t_1 \neq \mathrm{null}_t, \mathrm{train}(s) \neq t_1, \mathrm{alloc}(s_1) = \mathrm{idt}(t_1)$

$t \neq t_1, \mathrm{ids}(\mathrm{segm}(t)) < \mathrm{ids}(s_1), \mathrm{next}_t(t) = \mathrm{null}_t, \mathrm{alloc}(s_1) = \mathrm{tid}(t_1) \rightarrow \mathrm{next}'(t) = t_1 \wedge \mathrm{next}'(t_1) = \mathrm{null}_t$

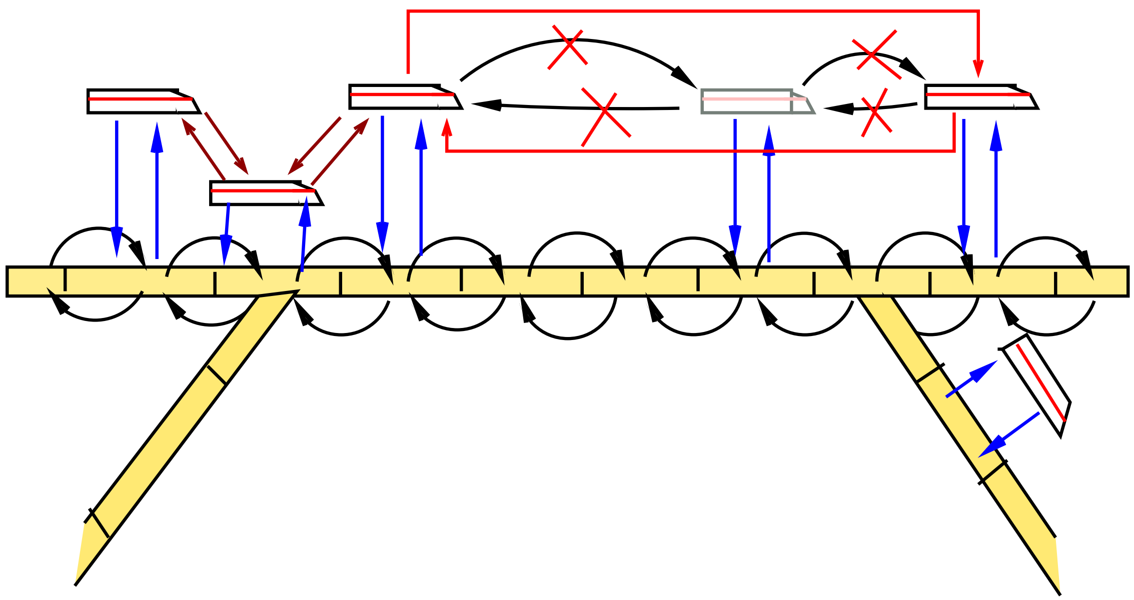
$t \neq t_1, \mathrm{ids}(\mathrm{segm}(t)) < \mathrm{ids}(s_1), \mathrm{alloc}(s_1) = \mathrm{tid}(t_1), \mathrm{next}_t(t) \neq \mathrm{null}_t, \mathrm{ids}(\mathrm{segm}(\mathrm{next}_t(t))) \leq \mathrm{ids}(s_1)$

$\quad \rightarrow \mathrm{next}'(t) = \mathrm{next}_t(t)$

...

$t \neq t_1, \mathrm{ids}(\mathrm{segm}(t)) \geq \mathrm{ids}(s_1) \rightarrow \mathrm{next}'(t) = \mathrm{next}_t(t)$

Incoming and outgoing trains



Safety property

Safety property we want to prove: no two trains ever occupy the same track segment:

$$(\text{Safe}) := \forall t_1, t_2 \;\; \text{segm}(t_1) = \text{segm}(t2) \rightarrow t_1 = t_2$$

In order to prove that (Safe) is an invariant of the system, we need to find a suitable invariant (Inv(i)) for every control location i of the TCS, and prove:

$$(\text{Inv}(i)) \models (\text{Safe}) \text{ for all locations } i$$

and that the invariants are preserved under all transitions of the system,

$$(\text{Inv}(i)) \wedge (\text{Update}) \models (\text{Inv}'(j))$$

whenever (Update) is a transition from location i to j .

Safety property

Need additional invariants.

- generate by hand [Faber, Ihlemann, Jacobs, VS, ongoing]

    use the capabilities of H-PILoT of generating counterexamples

- generate automatically [work in progress]

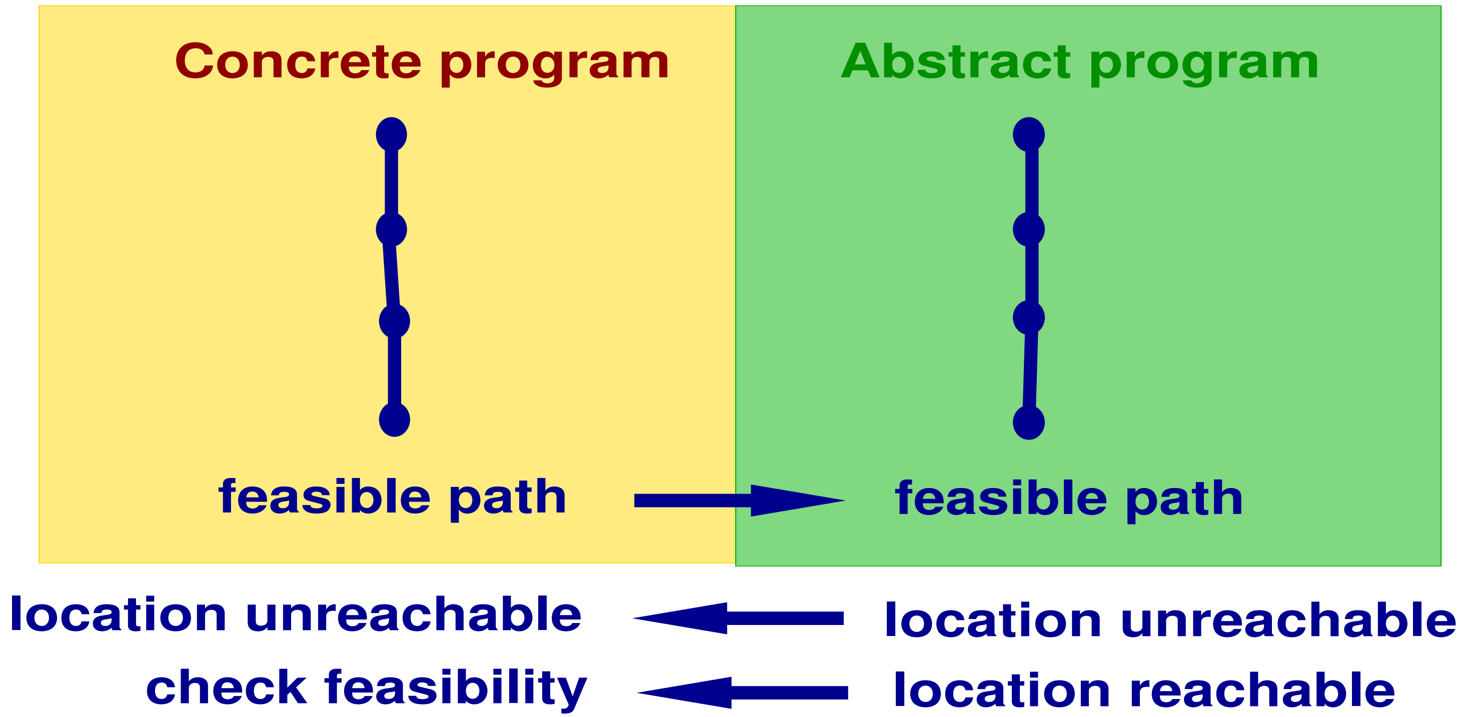
Ground satisfiability problems for pointer data structures

the decision procedures presented before can be used without problems

Other interesting topics

- Generate invariants
- Verification by abstraction/refinement

Abstraction-based Verification



$\Downarrow$

conjunction of constraints: $\phi(1) \wedge Tr(1,2) \wedge \cdots \wedge Tr(n-1,n) \wedge \neg \mathsf{safe}(n)$

- satisfiable: feasible path
- unsatisfiable: refine abstract program s.t. the path is not feasible

[McMillan 2003-2006] use 'local causes of inconsistency'

$\mapsto$ compute interpolants

Summary

- Decision procedures for various theories/theory combinations

  Implemented in most of the existing SMT provers:
  Z3: `http://z3.codeplex.com/`
  CVC4: `http://cvc4.cs.nyu.edu/web/`
  Yices: `http://yices.csl.sri.com/`

- Ideas about how to use them for verification

Decision procedures for other classes of theories/Applications"
Next semester: Seminar "Decision Procedures and Applications"

More details on Specification, Model Checking, Verification:
Every summer (usually end of August):
Summer school "Verification Technology, Systems & Applications"

BSc/MSc Theses in the area