Bradley & Manna "The Calculus of Computation"

Example 10.14 (pages 280–281)

$\mathcal{T}_1 = LI(\mathbb{Z})$ $\quad \mathcal{T}_2 = UIF_{\{f\}}$.

$$\varphi := 1 \leq x \wedge x \leq 2 \wedge f(x) \neq f(\underbrace{1}_{\omega_1}) \wedge f(x) \neq f(\underbrace{2}_{\omega_2})$$

1. Purification:

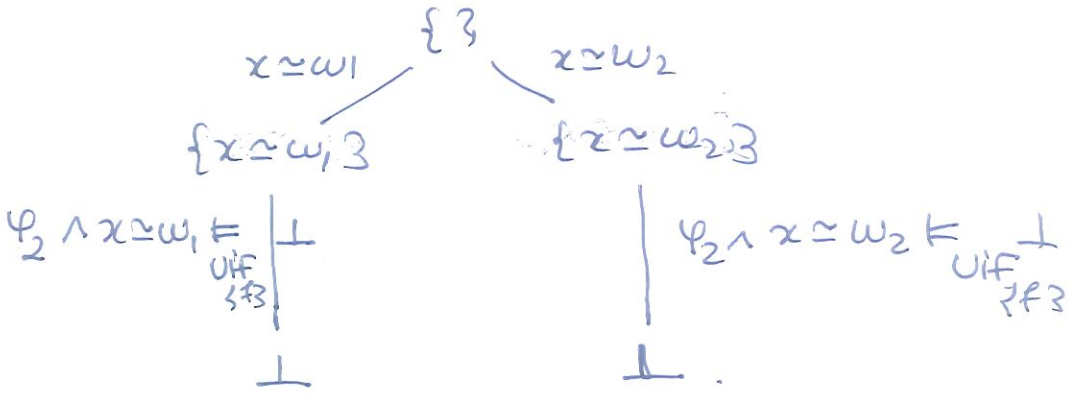
$LI(\mathbb{Z})$	$UIF_{\{f\}}$
$1 \leq x$	$f(x) \neq f(\omega_1)$
$x \leq 2$	$f(x) \neq f(\omega_2)$
$\omega_1 \simeq 1$	
$\omega_2 \simeq 2$	
(φ_1)	(φ_2)

$V := shared(\varphi_1, \varphi_2)$

$= \{x, \omega_1, \omega_2\}$.

2. Propagation

$\varphi_1 \models x \simeq \omega_1 \vee x \simeq \omega_2$ Case split on these two disjuncts



As all branches end in contradiction, φ is $\mathcal{T}_1 \cup \mathcal{T}_2$ unsatisf.

Cause of branching: non-convexity of $U(\mathbb{Z})$

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Example 10.13 (page 279)

$\mathcal{T}_1 = LI(\mathbb{Q})$ (convex) $\quad \mathcal{T}_2 = UIF_{\{f\}}$ (convex).

$$\varphi := f(\underbrace{f(x) - f(y)}_{w}) \neq f(z) \wedge x \leq y \wedge y + z \leq x \wedge 0 \leq z$$

($w = u - v$, with u = $f(x)$, v = $f(y)$)

1. Purification

$LI(\mathbb{Q})$	$UIF_{\{f\}}$
φ_1 sat. $\begin{cases} w = u - v \\ x \leq y \\ y + z \leq x \\ 0 \leq z \end{cases}$	φ_2 sat. $\begin{cases} f(w) \neq f(z) \\ u \simeq f(x) \\ v \simeq f(y) \end{cases}$

$\varphi_1 \models x \simeq y$ $\quad$ (*) $\quad x \simeq y$.

$u \simeq v$. $\quad$ (**) $\quad \models u \simeq v$

$\models z = w$ $\quad$ (***) $\quad z \simeq w$.

$\quad$ (⊥)

$V = \text{shared}(\varphi_1, \varphi_2) = \{x, y, z, u, v, w\}$.

Apply congruence closure on φ_2 : $\wedge x \simeq y$.

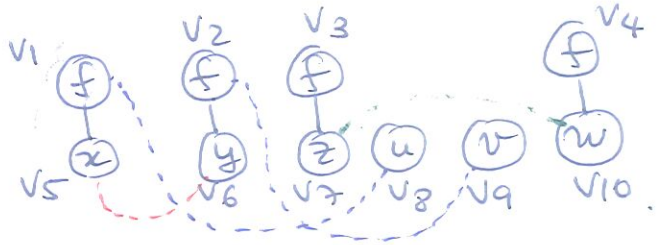
2. Propagation

$\varphi_1 \models x \simeq y$

follows from $\begin{cases} x \leq y \\ y + z \leq x \\ 0 \leq z \end{cases} \Rightarrow y \leq y + z \leq x$

Propagate (*).

- Check equalities between variables entailed by $\varphi_2 \wedge x \simeq y$
- → use congruence closure algorithm



$R = \{(V_1, V_8), (V_2, V_9), (V_5, V_6)\}$

Compute R^c

id	(*)	(**)	(***)
V_1	V_1 $\underline{V_8}$	V_8	V_8	V_9
V_2	V_2 V_2	V_6	V_9	V_9
V_3	V_3 V_3	V_3	V_3	V_3
V_4	V_4 V_4	V_4	V_4	V_4
V_5	V_5 V_5	V_5	$\underline{V_6}$	V_6
V_6	V_6 V_6	V_6	V_6	V_6
V_7	V_7 V_7	V_7	V_7	V_7
V_8	V_8 V_8	V_8	V_8	$\underline{V_9}$
V_9	V_9 V_9	V_9	V_9	V_9
V_{10}	V_{10} V_{10}	V_{10}	V_{10}	V_{10}

MERGE (V_1, V_8) $\quad$ FIND$(V_1) \neq$ FIND(V_8)

$P_{V_1} = P_{V_8} = \emptyset$.

UNION (V_1, V_8): FIND$(V_1) \leftarrow V_8$ (*)

MERGE (V_2, V_9) $\quad$ FIND$(V_2) \neq$ FIND(V_9)

$P_{V_2} = P_{V_9} = \emptyset$

UNION (V_2, V_9): FIND$(V_2) \leftarrow V_9$. (**)

MERGE (V_5, V_6) $\quad$ FIND$(V_5) \neq$ FIND(V_6)

$P_{V_5} = \{V_1\}$, $P_{V_6} = \{V_2\}$

UNION (V_5, V_6) $\quad$ FIND$(V_5) \leftarrow V_6$. (***)

MERGE (V_1, V_2) $\quad$ FIND$(V_1) \neq$ FIND(V_2)

$P_{V_1} = \emptyset = P_{V_2}$.

UNION (V_1, V_2) $\quad$ FIND$(V_1) :=$ FIND(V_2)

FIND$(V_8) =$ FIND(V_9)

$\Rightarrow \varphi_1 \wedge x \simeq y \models u \simeq v$ (UIF $\{f\}$)

Propagate (**).

$\varphi_2 \wedge x \simeq y \wedge u \simeq v \models z = w$

Propagate (***) $\quad R' = R \cup \{z \simeq w\}$.

↓