Universität Koblenz-Landau

FB 4 Informatik

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Collection of exercises: Part 1

Exercise 1. Let N be the following set of propositional clauses:

(1) $\neg P \vee Q \vee R$
(2) $Q \vee \neg T \vee \neg V \vee \neg W$
(3) $\neg P \vee Q \vee T \vee V \vee W$
(4) $P \vee \neg Q$
(5) $R \vee S$
(6) $R \vee \neg V$
(7) $\neg Q \vee T \vee \neg V \vee W$
(8) $\neg R \vee T$
(9) $\neg R \vee S$
(10) $\neg T \vee S \vee V \vee W$
(11) $\neg S \vee V$
(12) $\neg S \vee \neg T \vee \neg V \vee \neg W$
(13) $\neg V \vee W$

Use the DPLL method with backjumping to give a model. Use the DPLL inference rules with a reasonable strategy (i.e., use Fail or Backjump if possible, otherwise use Unit Propagate if possible, otherwise use Decide). If you use the Decide rule, use the largest undefined positive literal according to the ordering $P > Q > R > S > T > V > W$. If you use the Backjump rule, determine a suitable backjump clause using the 1UIP method and use the best possible successor state for that backjump clause.

Exercise 2. Assume $R \succ Q \succ P$. Let N_1 be the following set of clauses:

$$
\begin{array}{ll}
(C_1) & \neg R \vee \neg P \\
(C_2) & Q \vee P \\
(C_3) & \neg Q \\
(C_4) & R \vee \neg P \vee Q
\end{array}
$$

Use the ordered resolution calculus $\mathsf{Res}^{\succ}$ described in the lecture for checking the satisfiability of the set N_1 of clauses.

Exercise 3. Assume $P \succ Q \succ R \succ S$. Let N_2 be the following set of clauses:

$$\begin{array}{ll} (C_1) & \neg Q \vee \neg P \vee \neg S \\ (C_2) & R \vee P \\ (C_3) & Q \vee S \\ (C_4) & \neg R \vee S \end{array}$$

(1) Define a selection function S such that this set of clauses is saturated w.r.t. the ordered resolution calculus with selection $\mathsf{Res}^{\succ}_{S}$. Justify your choice.

(2) Sort the clauses according to $\succ_C$.

(3) Construct a model of N_2 using the canonical construction presented in the lecture.

Exercise 4. Give the definition of redundancy of a clause w.r.t. a set N of clauses.

Assume $P \succ S \succ Q \succ R$.

(1) Is the clause $P \vee \neg S$ redundant w.r.t. the set of clauses $\{\neg Q \vee P, R \vee \neg P, Q \vee \neg S\}$?

(2) Is the clause $\neg Q \vee R$ redundant w.r.t. the set of clauses $\{\neg Q \vee P, R \vee \neg P, Q \vee \neg S\}$?

Justify your answers.

Exercise 5. Let $\Sigma = (\{f/1, g/1, h/1, a\}, \{p/2, q/1, r/2\})$. Let X be a set of variables, and assume that $\{x, y, z, u, v, w, s, t\} \subseteq X$.

Let $\succ$ an ordering on ground atoms with the property that for all ground terms $t_1, ..., t_{12}$, $\neg p(t_1, t_2) \succ p(t_3, t_4) \succ \neg q(t_5, t_6) \succ q(t_7, t_8) \succ \neg r(t_9, t_{10}) \succ r(t_{11}, t_{12})$.

Let N be the following set of clauses:

$$\begin{array}{ll} (1) & \neg r(f(x), y) \vee p(g(x), x) \\ (2) & \neg q(h(g(z))) \vee \neg p(z, u) \\ (3) & q(h(v)) \\ (4) & r(w, g(s)) \vee p(t, f(s)) \end{array}$$

Use the ordered resolution calculus $\mathsf{Res}^{\succ}$ described in the lecture for checking the satisfiability of the set N of clauses.

Exercise 6. Consider the following formulae over a signature containing function symbols $\Omega = \{c/0, f/1\}$ and predicate symbols $\Pi = \{p/1\}$:

- $F_1 := p(c)$
- $F_2 := \forall x (p(x) \rightarrow p(f(x)))$
- $F_3 := p(f(f(f(c))))$.

Use resolution to prove that $\{F_1, F_2\} \models F_3$.

Exercise 7. Let $\Sigma = (\Omega, \Pi)$ with $\Omega = \{b/0, a/0, f/1\}$ and $\Pi = \{p/1\}$.

(1) How many Herbrand interpretations over Σ do exist? Explain briefly.

(2) How many Herbrand models over Σ has the following formula F?

$$F := p(b) \wedge \forall x \neg p(f(f(f(x))))$$

Justify your answer.

Exercise 8. Let $\Sigma = (\Omega, \Pi)$ with $\Omega = \{a/0, b/0\}$ and $\Pi = \{p/2, q/1\}$.

(1) How many Herbrand interpretations over Σ exist? Explain briefly.

(2) How many Herbrand models over Σ has the following formula F?

$$F := \neg p(a, b) \wedge q(a) \wedge p(a, a)$$

Justify your answer.

Exercise 9.

(a) Give definitions for the following fragments of first-order logic:
- The Bernays-Schönfinkel class;
- The Ackermann class.
- The monadic class.

(b) What is the idea in the proof of decidability for the Bernays-Schönfinkel class?

(c) To which of these classes do the following formulae belong (note that they can be in more than one, or in none of the classes above):

(1) $\exists y \forall x \ \ ((p(x) \vee r(x, y)) \wedge q(y))$

(2) $\forall x \exists y \exists u ((p(x) \vee q(y)) \wedge (q(y) \vee p(u)))$

(3) $\exists z \forall x \exists y (p(x) \vee q(y)) \wedge q(z)$

(4) $\exists x \forall y ((p(x) \vee r(y)) \wedge q(y))$

(5) $\forall x \exists y \forall z \exists u ((p(x) \vee s(x, y, z)) \wedge (q(y) \vee p(u) \vee s(x, z, u)))$

(6) $\exists z \forall x \exists y ((p(x) \vee r(x, y)) \wedge q(z))$