Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans **November 24, 2022**

Exercises for "Decision Procedures for Verification"
Exercise sheet 1

Exercise 1.1: (*5 P*)
Determine which of the following formulas are valid/satisfiable/unsatisfiable:

(1) $(P \wedge Q) \rightarrow (P \vee Q)$
(2) $(P \vee Q) \rightarrow (P \wedge Q)$
(3) $\neg(P \wedge \neg\neg P)$
(4) $Q \rightarrow \neg Q$
(5) $Q \wedge \neg Q$
(6) $\neg(\neg P \vee \neg\neg P)$
(7) $\big((P \rightarrow Q) \wedge (\neg P \rightarrow R)\big) \rightarrow (Q \vee R)$

Exercise 1.2: (*5 P*)
If F and G are propositional formulae then prove that the following are equivalent:

(a) $F \models G$;

(b) $\models F \rightarrow G$ (i.e. $F \rightarrow G$ is valid);

(c) $F \wedge \neg G$ is unsatisfiable.

Exercise 1.3: (*2 P*)
Prove: If N is a set of propositional formulas, then $N \models F$ if and only if $N \cup \{\neg F\}$ is unsatisfiable.

(A set of propositional formulas is unsatisfiable, if and only if for every valuation $\mathcal{A}$ there is a formula G in the set such that $\mathcal{A} \not\models G$.)

Exercise 1.4: (*5 P*)
Prove:

(1) If $F_1, \ldots, F_n, G$ are propositional formulae then $F_1 \wedge \cdots \wedge F_n \rightarrow G$ is valid iff every valuation which is a model of all the formulae $F_1, F_2, \ldots F_n$ is also a model of G.

(2) If $F_1, \ldots, F_n, G$ are propositional formulae then the following are equivalent:

   (a) $\{F_1, \ldots, F_n\} \models G$
   (b) $F_1 \wedge \cdots \wedge F_n \to G$ is valid
   (c) $F_1 \wedge \cdots \wedge F_n \wedge \neg G$ is unsatisfiable.

Supplementary exercises

Exercise 1.5: *(2 P)*

Consider the formulae $F_n = \bigvee_{i=1}^{n} (Q_i \wedge R_i)$ for $n \in \mathbb{N}$.
As a function of n, how many clauses are in:

(1) the CNF formula F' constructed using the distributivity of disjunctions over conjunctions?
(2) the CNF formula F'' obtained using the satisfiability-preserving translation to clause form?
(3) For which n is the first approach better?

Please submit your solution until Monday, 31.10.2022 at 16:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- Homework option in the Course in OLAT.