Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans **January 10, 2023**

Exercises for "Decision Procedures for Verification"
Exercise sheet 10

Exercise 10.1: *(4 P)*

(I) Let F_1 be the following conjunction (in linear rational arithmetic $LI(\mathbb{Q})$):

$$F_1: \quad x_1 + x_2 + 2x_3 = 2 \ \wedge \ x_1 + x_3 + \frac{1}{5} < 0 \ \wedge \ x_2 - x_3 \leq \frac{1}{2} \ \wedge \ x_1 + 5x_3 \leq 5$$

Check the satisfiability of F_1 using the Fourier-Motzking method for quantifier elimination for the formula $\exists x_3 \exists x_2 \exists x_1 F_1$.

(II) Consider the following formulae (in linear rational arithmetic $LI(\mathbb{Q})$):

$$F_2: \quad \exists x \forall y \exists z (y > 0 \vee (x + y - z < 0 \wedge x + y + z < 0))$$
$$F_3: \quad \forall x \exists y \exists z (2x - y > 0 \wedge 2y - z > 0 \wedge z - y \geq 2 \wedge x - y < 0 \wedge y < 0)$$

Check whether F_2 and F_3 are valid or satisfiable using the Fourier-Motzkin method for quantifier elimination.

Hint: *You can eliminate the quantifiers one after the other, starting with the innermost quantifier. To eliminate universal quantifiers you can proceed as follows:*

- *use the equivalence:* $\forall x G(x) \equiv \neg \exists x \neg G(x)$;
- *compute a DNF for* $\neg G(x)$, *i.e. find conjunctions of literals* $G_1, \dots, G_k$ *such that* $\neg G(x) \equiv G_1 \vee \dots \vee G_k$;
- $\neg \exists x \neg G(x) \equiv \neg \exists x \ (G_1 \vee \dots \vee G_n) \equiv \neg(\exists x G_1 \vee \dots \vee \exists x G_n)$;
- *apply the Fourier-Motzkin method for quantifier elimination for* $\exists x G_1, \dots, \exists x G_n$.

Exercise 10.2: *(2 P)*
Let F_1 be the following conjunction (in linear rational arithmetic $LI(\mathbb{Q})$):

$$F_1: \quad x_1 + x_2 + 2x_3 = 2 \ \wedge \ x_1 + x_3 + \frac{1}{5} < 0 \ \wedge \ x_2 - x_3 \leq \frac{1}{2} \ \wedge \ x_1 + 5x_3 \leq 5$$

Check the satisfiability of F_1 using the Loos-Weispfenning method for quantifier elimination.

Please submit your solution until Tuesday, January 17, 2023 at 17:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities: Use the directory Homework 10 in OLAT;