

Exercises for “Decision Procedures for Verification” Exercise sheet 12

Exercise 12.1: (2 P)

Let \mathcal{T} be the combination of $LI(\mathbb{Q})$ (linear arithmetic over \mathbb{Q}) and UIF_{Σ} , the theory of uninterpreted function symbols in the signature $\Sigma = \{\{f/1, g/2\}, \emptyset\}$.

Check the satisfiability of the following ground formula w.r.t. \mathcal{T} using the deterministic version of the Nelson-Oppen procedure (after purifying the formulae check for entailment of equalities between shared constants and propagate the entailed equalities):

$$\phi := (g(c + d, e) \approx f(g(c, d)) \wedge c + e \approx d \wedge e \geq 0 \wedge c \geq d \wedge g(c, c) \approx e \wedge f(e) \not\approx g(c + c, 0))$$

Exercise 12.2: (4 P)

Use the Nelson-Oppen procedure for checking the satisfiability of the following formulae:

(1) $1 \leq c \wedge c \leq 3 \wedge f(c) \not\approx f(1) \wedge f(c) \not\approx f(3) \wedge f(1) \not\approx f(2)$
in the combination $LI(\mathbb{Z}) \cup UIF_{\{f\}}$.

(2) $f(c) \approx f(c + d) \wedge 1 \leq c \wedge c \leq d + e \wedge c + e \leq d \wedge d = 1 \wedge f(c) \not\approx f(2)$
in the combination $LI(\mathbb{Z}) \cup UIF_{\{f\}}$.

Exercise 12.3: (2 P)

Check the satisfiability w.r.t. $\mathcal{T} = LI(\mathbb{Q})$ of the following set of ground clauses using the “lazy” approach to SMT presented in the class.

$$(\neg(0 \leq x) \vee \neg(y \leq z)) \wedge (\neg(z \leq x + y) \vee (y \leq z)) \wedge (\neg(0 \leq y) \vee (0 \leq x)) \wedge (z \leq x + y)$$

For theory reasoning in $LI(\mathbb{Q})$ use the Fourier-Motzkin algorithm.

Supplementary exercises:

Exercise 12.4: (5 P)

We say that a theory \mathcal{T} is *stably infinite* if for every quantifier-free formula ϕ , ϕ is satisfiable in \mathcal{T} iff ϕ is satisfiable in a (countably) infinite model of \mathcal{T} .

Let $\mathcal{T}_1, \mathcal{T}_2$ be stably infinite theories with disjoint signatures. Prove that their combination $\mathcal{T}_1 \cup \mathcal{T}_2$ is stably infinite.

Please submit your solution until Tuesday, January 31, 2023 at 17:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities: Under HOMEWORK 12 in OLAT;