Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans

November 7, 2022

Exercises for "Decision Procedures for Verification"
Exercise sheet 3

Exercise 3.1: (*2 P*)
Assume $P \succ Q \succ R$. Let N be the following set of clauses:

$$\begin{array}{ll} (1) & \neg R \vee P \\ (2) & \neg Q \vee \neg Q \vee \neg P \\ (3) & Q \\ (4) & R \vee P \end{array}$$

Let S be the selection function which selects $\neg R$ in clause (1) and both occurrences of $\neg Q$ in clause (2).

Use the ordered resolution calculus with selection $\mathsf{Res}^{\succ}_{S}$ described in the lecture for checking the satisfiability of the set N of clauses.

Exercise 3.2: (*2 P*)
A propositional Horn clause is a clause which has at most one positive literal.
(*Example:* $\neg P \vee Q \vee \neg R$, $\neg P \vee \neg R$ and Q are Horn clauses,
whereas $\neg P \vee Q \vee R$ and $Q \vee R$ are not Horn clauses.)
Prove: Every set H of clauses with the following properties:

(i) H consists only of Horn clauses;

(ii) Every clause in H contains at least one negative literal;

is satisfiable.

Exercise 3.3: (*5 P*)
Let H be a set of propositional Horn clauses. The size of H is the number of all literals which occur in H. Prove that the resolution calculus $\mathsf{Res}^{\succ}_{S}$ (for a suitable selection function S) can check the satisfiability of H in time polynomial in the size of H.

Hint: With which choice of the selection function can one model the marking algorithm discussed in the lecture "Logik für Informatiker"?

Supplementary question (will be discussed during the exercise session):
Can you give an algorithm for check the satisfiability of H in time linear in the size of H?

Exercise 3.4: (*2 P*)
Use a DPLL procedure to find a model of each of the following formulae, or prove that the particular formula has no model:

(1) $(P \vee \neg Q) \wedge (\neg P \vee Q) \wedge (Q \vee \neg R) \wedge (\neg Q \vee \neg R)$
(2) $(P \vee Q \vee \neg R) \wedge (P \vee \neg Q) \wedge (P \vee Q \vee R) \wedge (R \vee Q) \wedge (R \vee \neg Q) \wedge (\neg P \vee \neg R) \wedge \neg U$

Please submit your solution until Tuesday, November 15, 2022 at 17:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- Use the Homework 03 folder in OLAT