Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Decision Procedures for Verification"
Exercise sheet 5

Exercise 5.1: *(3 P)*
Let $\Sigma = \{0, s, +\}$. Consider the following formulae in the signature Σ:

1. $F_1 = \forall x\ (x + 0 \approx x)$
2. $F_2 = \forall x, y\ (x + s(y) \approx s(x + y))$
3. $F_3 = \forall x, y\ \ (x + y \approx y + x)$.

Find a Σ-structure in which F_1 and F_2 are valid but F_3 is not.

Exercise 5.2: *(2 P)*
Compute a clausal normal form for the following formula:

$$\exists x\, \forall y\, (\forall z\, (p(y, z) \vee \neg\, x \approx y) \rightarrow (\forall z\, q(y, z) \wedge \neg\, r(x, y)))$$

Exercise 5.3: *(4 P)*
Let $\Sigma = (\Omega, \Pi)$ with $\Omega = \{b/0, f/1\}$ and $\Pi = \{p/1\}$.

(a) Which is the universe of the Herbrand interpretations over this signature?
If $\mathcal{A}$ is a Herbrand interpretation over Σ how are $b_{\mathcal{A}}$ and $f_{\mathcal{A}}$ defined?

(b) How many different Herbrand interpretations over Σ do exist? Explain briefly.

(c) How many different Herbrand models over Σ does the formula:

$$p(f(f(b))) \wedge \forall x(p(x) \rightarrow p(f(x))) \qquad (1)$$

have? Explain briefly.

(d) Every Herbrand model over Σ of (**??**) is also a model of

$$\forall x\ p(f(f(x))) \qquad (2)$$

Give an example of an algebra that is a model of (1) but not of (2).

Exercise 5.4: *(1 P)*
Which of the following formulae is in the Bernays-Schönfinkel class?

(1) $\exists y \forall x \exists z \quad ((p(x) \vee q(y)) \wedge (p(z) \vee \neg q(y))$

(2) $\forall x \exists y \forall z \exists u((p(x) \vee q(y)) \wedge (q(y) \vee r(u, x))$

(3) $\exists y \exists z \forall x[(p(x) \vee q(y)) \wedge q(z)]$

Supplementary exercise

Exercise 5.5: *(4 P)*
Let $\Sigma = (\Omega, \Pi)$ be a signature and X a set of variables. Let $\mathcal{A}$ be a Σ-structure and $\beta : X \to U_{\mathcal{A}}$ a variable assignment.

(1) Prove that for every formula $F \in F_\Sigma(X)$ and every $x \in X$, the truth values $\mathcal{A}(\beta)(\forall x F)$ and $\mathcal{A}(\beta)(\exists x F)$ do not depend on $\beta(x)$.

(2) Use (1) to show that if G is a closed formula in $F_\Sigma(X)$, then the truth value of G in $\mathcal{A}$ w.r.t. β, $\mathcal{A}(\beta)(G)$ does not depend on the way β is defined.

(3) Use (2) to prove that $\mathsf{Th}(\mathsf{Mod}(\mathcal{F})) = \{G \in F_\Sigma(X) \text{ closed} \mid \mathcal{F} \models G\}$.

Please submit your solution until Tuesday, November 29, 2022 at 17:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

- Use the folder Homework 05 in OLAT