Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans

November 29, 2022

Exercises for "Decision Procedures for Verification"
Exercise sheet 6

Exercise 6.1: *(2 P)*
Let $\Sigma = (\Omega, \Pi)$ with $\Omega = \{a/0, b/0, f/1\}$ and $\Pi = \{p/1\}$.

(1) Which is the universe of the Herbrand interpretations over this signature? How are the function symbols defined?

(2) How many different Herbrand interpretations over Σ exist? Explain briefly.

(3) How many Herbrand models over Σ has the formula F below?

$$F := p(b) \wedge \forall x\ \neg p(f(f(x)))$$

Justify your answer.

Exercise 6.2: *(2 P)*
Let $\Sigma = (\Omega, \Pi)$ with $\Omega = \{a/0, b/0, c/0\}$ and $\Pi = \{p/1, q/1\}$.

(1) How many Herbrand interpretations over Σ exist? Explain briefly.

(2) How many Herbrand models over Σ has the following formula F?

$$F := \neg p(a) \wedge p(b) \wedge q(c) \wedge \neg q(a)$$

Justify your answer.

Exercise 6.3: *(2 P)*
Compute a most general unifier of

$$\{\, f(x, g(x)) = y,\ h(y) = h(v),\ v = f(g(z), w) \,\}$$

using the method presented in the lecture.

Exercise 6.4: *(L P)*
et $\Sigma = (\Omega, \Pi)$, where $\Omega = \{a/0, f/1, g/1\}$ and $\Pi = \{p/2\}$.

Use the resolution calculus Res described in the lecture to show that the following set of clauses (where x, y, z are variables) is unsatisfiable:

$$p(a, z)$$
$$\neg p(f(f(a)), a)$$
$$\neg p(x, g(y)) \vee p(f(x), y)$$

For computing the most general unifiers use the method presented in the lecture.

Exercise 6.5: *(3 P)*
Consider the following formulae:

- $F_1 := \forall x(S(x) \rightarrow \exists y(R(x, y) \wedge P(y)))$
- $F_2 := \forall x(P(x) \rightarrow Q(x))$
- $F_3 := \exists x S(x)$
- $G := \exists x \exists y(R(x, y) \wedge Q(y))$

Use the resolution calculus to prove that $\{F_1, F_2, F_3\} \models G$.

Please submit your solution until Tuesday, December 6, 2022 at 17:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- Use the folder Homework 06 in OLAT