Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Decision Procedures for Verification"
Exercise sheet 7

Exercise 7.1: *(3 P)*
Let $\succ$ be a total and well-founded ordering on ground atoms such that, if the atom A contains more symbols than B, then $A \succ B$. Let N be the following set of clauses:

$$\neg q(z, z)$$
$$\neg q(f(x), y) \vee q(f(f(x)), y) \vee p(x)$$
$$\neg p(a) \vee \neg p(f(a)) \vee q(f(a), f(f(a)))$$
$$p(f(x)) \vee p(g(y))$$
$$\neg p(g(a)) \vee p(f(f(a)))$$

(a) Which literals are maximal in the clauses of N?
(b) Define a selection function S such that N is saturated under $Res_S^{\succ}$. Justify your choice.

Exercise 7.2: *(3 P)*
Let $\Sigma = (\{f/1, h/1\}, \{p/2, q/2, r/1\})$. Let X be a set of variables, and $\{x, y\} \subseteq X$.

Let $\succ$ an ordering on ground atoms with the property that for all ground terms $t_1, ..., t_{12}$, $\neg p(t_1, t_2, t_3) \succ p(t_4, t_5, t_6) \succ \neg q(t_7, t_8) \succ q(t_9, t_{10}) \succ \neg r(t_{11}) \succ r(t_{12})$.

Let N be the following set of clauses:

(1)	$r(h(x)) \vee r(y)$
(2)	$\neg q(f(x), y) \vee p(x, x)$
(3)	$\neg r(h(f(x))) \vee \neg p(x, y)$
(4)	$q(y, x) \vee p(y, x)$

Use the ordered resolution calculus $\mathsf{Res}^{\succ}$ described in the lecture for checking the satisfiability of the set N of clauses.

Exercise 7.3: *(2 P)*
Let F and G be propositional formulae over $\Pi = \{P, Q, R, S, T, U\}$ such that:

- The CNF of F is the following set N of clauses:

(1)	$P \vee Q$
(2)	$\neg P \vee R \vee S$
(3)	$\neg P \vee \neg R$
(4)	$P \vee U$

- The CNF of $\neg G$ consists of the set M of clauses:

$$\begin{array}{ll} (5) & R \vee \neg S \\ (6) & \neg R \vee Q \\ (7) & \neg Q \vee R \\ (8) & \neg S \vee T \\ (9) & S \vee \neg T \\ (10) & \neg Q \vee \neg R \end{array}$$

Which propositional variables occur only in N and not in M?

Which propositional variables occur both in N and in M?

Use the method described in the lecture to construct a Craig interpolant for $F \models G$.

Exercise 7.4: (*5 P*)
Assume $S \succ P \succ Q \succ R$. Let N be the following set of clauses:

$$\begin{array}{ll} (1) & \neg Q \vee P \vee R \\ (2) & \neg R \vee P \\ (3) & Q \vee S \vee \neg P \\ (4) & \neg Q \vee \neg S \end{array}$$

Give the definition of redundancy of a clause w.r.t. a set of clauses.
Is the clause $\neg Q \vee P \vee S$ redundant w.r.t. the set N above?
Is the clause $\neg Q \vee P$ redundant w.r.t. the set N above? Justify your answer.

Assume $U \succ S \succ P \succ Q \succ R$. Let N be the following set of clauses:

$$\begin{array}{ll} (1) & \neg Q \vee P \vee R \\ (2) & \neg R \vee P \\ (3) & \neg Q \vee P \vee S \\ (4) & Q \vee S \vee \neg P \\ (5) & \neg Q \vee \neg S \end{array}$$

Is the clause $\neg Q \vee P \vee S \vee U$ redundant w.r.t. the set consisting of the clauses (1), (2), (4) and (5)? Justify your answer.

Supplementary exercise

Exercise 7.5: (*2 P*)
Redundant clauses remain redundant, if the theorem prover deletes redundant clauses. Prove: If N and M are sets of clauses and $M \subseteq \text{Red}(N)$, then $\text{Red}(N) \subseteq \text{Red}(N \backslash M)$.

Please submit your solution until Tuesday, December 13, 2022 at 17:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- Use the folder Homework 07 in OLAT