

### Exercises for “Decision Procedures for Verification” Exercise sheet 9

#### Exercise 9.1: (2 P)

Check the satisfiability of the following ground formula using the algorithm based on congruence closure presented in the lecture.

- $\phi = h(c, e) \approx d \wedge g(d) \approx e \wedge g(h(c, g(d))) \not\approx e$ .

You will be able to solve the following exercises after the lecture on Monday, 9. 01. 2023.

#### Exercise 9.2: (6 P)

Check the satisfiability of the following formulae in positive difference logic w.r.t.  $\mathbb{Q}$ ; in case of satisfiability find a satisfying assignment.

- (1)  $x - y \leq 3 \wedge y - z \leq 2 \wedge x - z \leq 1 \wedge x - u \leq -3$ .
- (2)  $x - y \leq 3 \wedge y - z \leq 2 \wedge x - z \leq 1 \wedge x - u \leq -3 \wedge u - x \leq 1$ .
- (3)  $x - y \leq 3 \wedge y - z \leq 2 \wedge x - z \leq 1 \wedge x - u \leq -3 \wedge u - z \leq 3 \wedge z - x \leq 1$ .

#### Exercise 9.3: (4 P)

(I) Check the satisfiability of the following conjunctions in difference logic w.r.t.  $\mathbb{Z}$ ; in case of satisfiability find a satisfying assignment.

- (1)  $x - y < 4 \wedge y - z \leq 2 \wedge z - x < -3 \wedge x - u \leq -3$ .
- (2)  $x - y < 4 \wedge y - z \leq 2 \wedge z - x \leq -5 \wedge x - u < -3 \wedge u - x \leq 4$ .
- (3)  $x - y < 4 \wedge y - z \leq 2 \wedge z - x < -5 \wedge x - u < -3 \wedge u - x \leq 4$ .

(II) Check the satisfiability of the following formulae in difference logic w.r.t.  $\mathbb{Q}$ ; in case of satisfiability find a satisfying assignment.

- (1)  $x - y < 4 \wedge y - z \leq 2 \wedge z - x < -5 \wedge x - u \leq -3$ .
- (2)  $x - y < 4 \wedge y - z \leq 2 \wedge z - x \leq -6 \wedge x - u \leq -4 \wedge u - x \leq 4$ .
- (3)  $x - y < 4 \wedge y - z \leq 2 \wedge z - x \leq -7 \wedge x - u < -3 \wedge u - x \leq 4$ .

*Hint:* It is sufficient to check the existence of negative cycles in the associated graphs by looking at the graphs; in this assignment you do not have to use the Bellman-Ford algorithm for this.

Please submit your solution until Wednesday, January 11, 2023 at 16:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- Use the directory Homework 9 in OLAT.